

Module-04

Design of a General Purpose Industrial Helical Gear Reduction Unit- Part II

Lecture-16

Bearing Arrangement-Gear Box Shafts



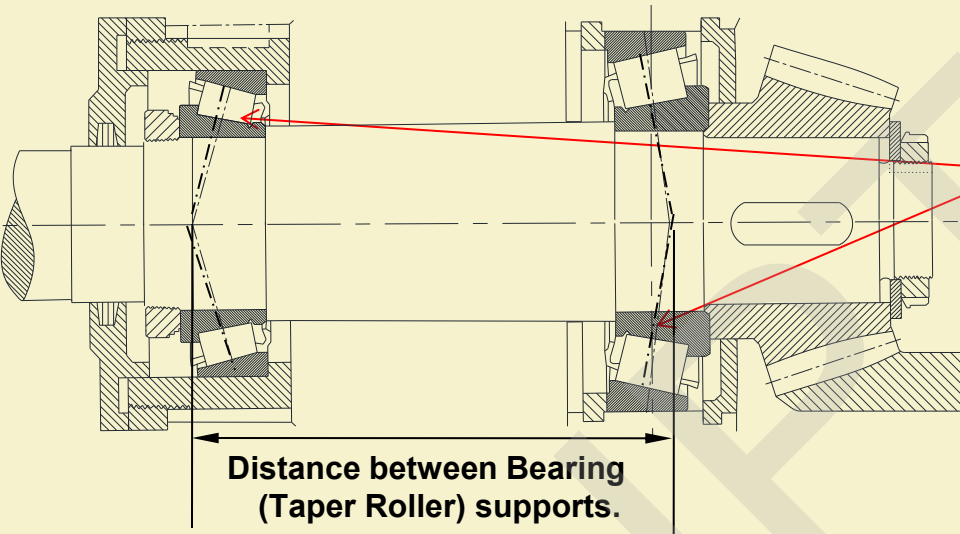
Outline of the Lecture

- **Rolling Element Bearing Arrangement on Shaft**
 - **Load Centers for Bearing Support**
 - **Layout of Single Stage Gear Box**
 - **Layout of Intermediate Shaft of Designed Gear Box, Gear Distance and Bearing support distance**
 - **Bearing Life Estimation, Consideration of different Factors**
 - **Typical Bearing Dimensions and Load Carrying Capacity (Extract from Catalogue)**
- **Steps in Bearing Load Estimation.**
 - **Bearing Life Estimation in Revolution and Hours.**
 - **Loads on Shaft from Gears (Recapitulation).**

Bearing Arrangements in Gear Box Shafts :

Step-5

In Layout of pinion and gears shafts are automatically shaped.



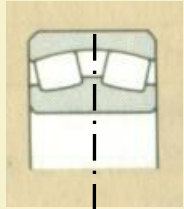
Then Bearing types are chosen taking into account service severity and life.

**Taper Roller Bearing to be used in pair.
Other Bearings- may be used in pair
or in combination.**

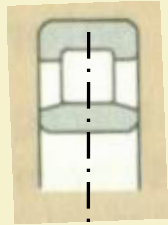
For an example both spherical roller and ball bearing can be combined with cylindrical roller bearing in the other end.

Bearing Arrangements

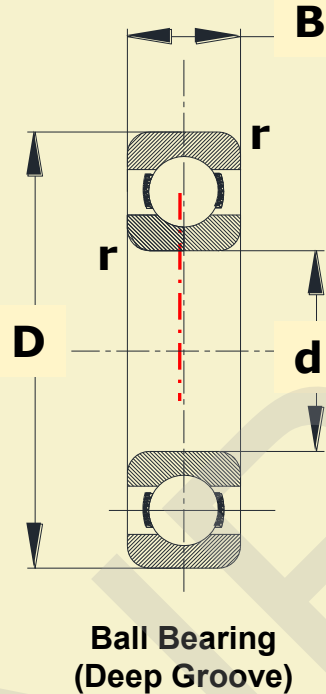
Step-5



Spherical Roller Bearing



Cylindrical Roller Bearing



Ball Bearing
(Deep Groove)

Choice depends on type of loading mainly.

Locking of bearings with shaft and housing is to be decided at this stage.

Sharing of reaction loads by bearings depends also on of bearing Locking arrangement.

With distance between bearing supports the shaft is considered *as "simply supported beam"*.

Step 5:

Technical drawing of a shaft-hub assembly. The drawing shows a shaft with a keyway and a hub with a keyway. The dimensions are labeled as follows:

- β : Angle of the keyway.
- Z_p : Distance from the shaft centerline to the keyway.
- Z_g : Distance from the hub centerline to the keyway.
- b : Width of the hub.
- A : Total length of the assembly.

Layout in Step 3 & Step



Layout & Preliminary Bearing Selection (Contd.....)

Step 3 to 5:

Roller Bearings:

15 to 25 mm
(Typical)

10 to 15 mm (Typical)

15 to 25 mm
(Typical)

(Not taper roller
in this case).

Considering
bearing widths,

finally distances
are marked.

53

50

178

Layout of Intermediate Shaft (Referring to Design Problem)

Bearing (Rolling Element) Selection

Life Estimation of Rolling Element Bearing

Equivalent Load (P) Acting on bearing

$$P = C_1 (XVF_r + YF_a)$$

Where,

C_1 = A factor on nature of shock load

V = Outer race rotation factor (1 for fixed & 1.2 for rotating)

F_r = Net radial load acting on bearing

F_a = Net axial load acting on bearing

X = Radial Load Factor (From Catalogue)

Y = Axial Load Factor " "

Sample Data from Bearing Catalogue (SKF)**Recommended X & Y Factors – *Contd.***

$$P = C_1 (XVF_r + YF_a)$$

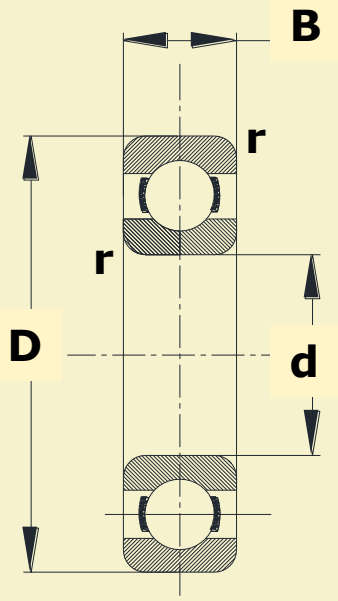
When C/P =	4	8	16
Y=	1.3	1.6	2

Bearing Type	Factor X		Factor- Y
	Rotating inner-ring load	Stationary inner-ring load	
Single-row deep groove ball bearing, series EL, R, 60X, 62, 63, 64, RLS, RMS, EE.	1	1.4	1.6

When the value of P is less than $1.4F_r$, then use $P = 1.4F_r$

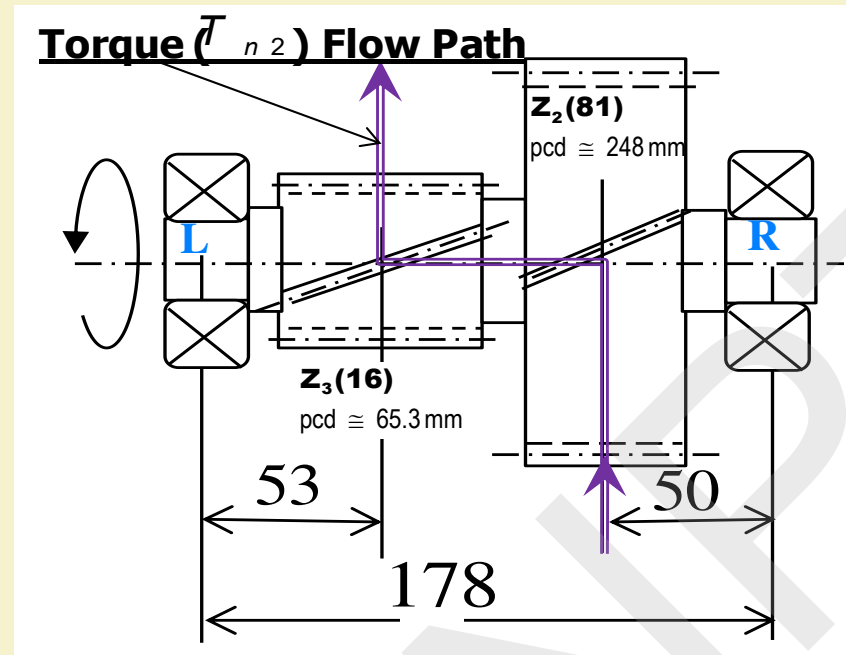
Sample Ball Bearing Data (SKF)

Bearing No.	d		D		B		r	Basic Capacity, lb.	
	mm.	in.	mm.	in.	mm.	in.		Dynamic C	Static C ₀
6300	10	0.3937	35	1.3870	11	0.4331	1	1430	800
01	12	0.4724	37	1.4567	12	0.4724	1.5	1760	950
02	15	0.5906	42	1.6535	13	0.5118	1.5	1930	1140
6303	17	0.6693	47	1.8504	14	0.5512	1.5	2320	1370
04	20	0.7874	52	2.0472	15	0.5906	2	2750	1700
05	25	0.9843	62	2.4409	17	0.6693	2	3600	2280
6306	30	1.1811	72	2.5346	19	0.7480	2	4800	3200
07	35	1.3780	80	3.1496	21	0.8268	2.5	5700	3800
08	40	1.5748	90	3.5433	23	0.9055	2.5	6950	4800
6309	45	1.7717	100	3.9370	25	0.9843	2.5	9000	6550
10	50	1.9685	110	4.3307	27	1.0630	3	10400	7800
11	55	2.1654	120	4.7244	29	1.1417	3	11800	9300



Bearing Life Estimation

For Intermediate Shaft (Referring to Design Problem)

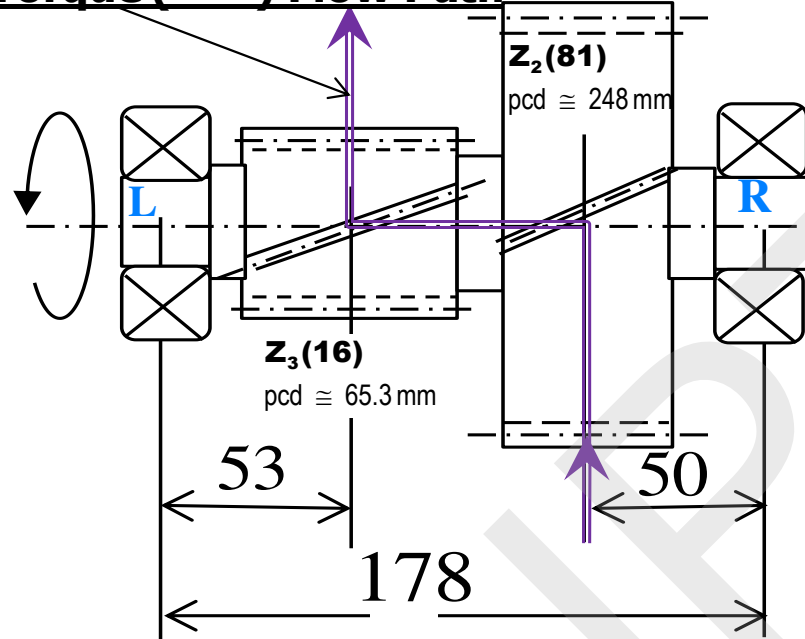


Intermediate Shaft with gears and Bearings (Plan View)

An intermediate shaft transmits power (torque & motion) from previous shaft to the next shaft.

The shaft is supported by bearings. Teeth contacts experience forces due to torque. Reaction loads are taken by the support bearings.

Shaft is subjected to bending moments. Once the bearing positions are finalized the reaction loads are calculated and equivalent loads on bearing and bearing life is estimated.

For Intermediate Shaft (Referring to Design Problem)**Torque (T_{n2}) Flow Path****Intermediate Shaft with gears and Bearings (Plan View)**

Equivalent Load Acting on bearing is expressed as:

Life of Rolling Element bearing in Number of Revolution is expressed as:

$$P = C_1 (XV F_r + YF_a)$$

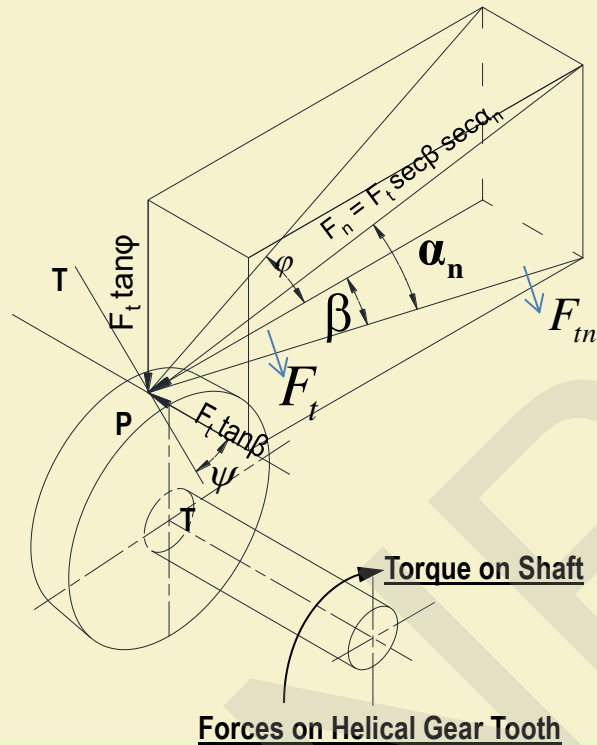
$$L_N = \left(\frac{C}{P} \right)^\varepsilon \times 10^6 \quad \text{Revolution}$$

$\varepsilon = 3$ for Ball Bearing & $\frac{10}{3}$ for Roller Bearing

Life in hours is then estimated as:

$$L_H = \frac{L_N}{N \times 60} \quad \text{Hours}$$

4th. Step. Loads on Gear, Pinion Teeth and on Bearings.



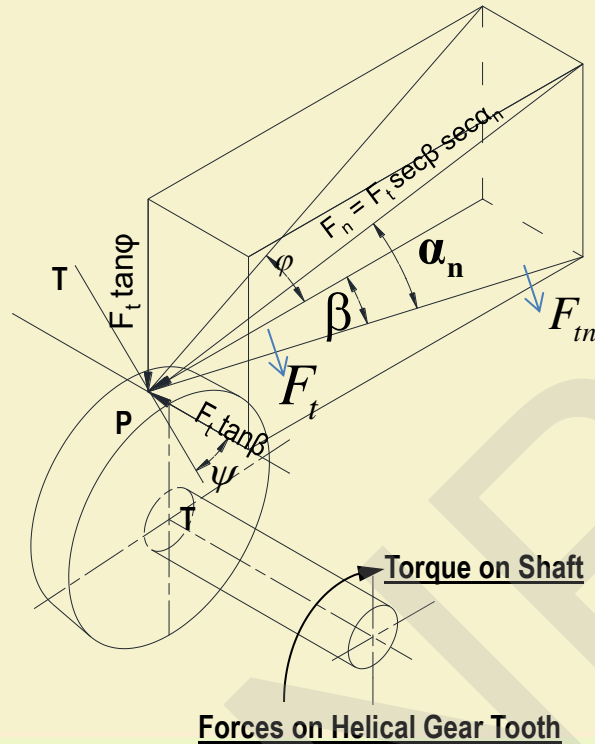
Tangential Load: $F_t = \frac{2T}{d_p}$

Where, $\left(d_p = \frac{Z \times m_n}{\cos \beta} \right)$

Normal Load: $F_{tn} = \frac{F_t}{\cos \beta}$

$$F_n = \frac{F_{tn}}{\cos \alpha_n} = F_t \sec \beta \cdot \sec \alpha_n$$

4th. Step. Loads on Gear, Pinion Teeth and on Bearings.



Radial Load:

$$F_r = F_n \cdot \sin \alpha_n$$

$$= F_t \sec \beta \cdot \sec \alpha_n \cdot \sin \alpha_n$$

$$F_r = F_t \sec \beta \cdot \tan \alpha_n$$

$$(= F_t \tan \varphi)$$

Axial Load:

$$F_a = F_n \sin \beta = F_t \tan \beta$$

Thank You



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Prof. Rathindranath Maiti
Mechanical Engineering Department

Module-04

Design of a General Purpose Industrial Helical Gear Reduction Unit- Part II

Lecture-17

Bearing Load Calculation- Intermediate Shaft



Outline of the Lecture

- **Loads on Shaft from 1st. Stage Gear & 2nd. Stage Pinion**
 - **Bearing Reactions**
 - **Direction of Helix on Pinion and Gear to Minimize Axial Load Reactions**
 - **Additional Bending Moment on Shaft Due to axial Loads**
- **Resultant Bearing Reactions at Two Bearing Act at Different Planes.**

**Layout & Preliminary
Bearing Selection** (Recapitulation)

Step 3 to 5:

Roller Bearings:

15 to 25 mm
(Typical)

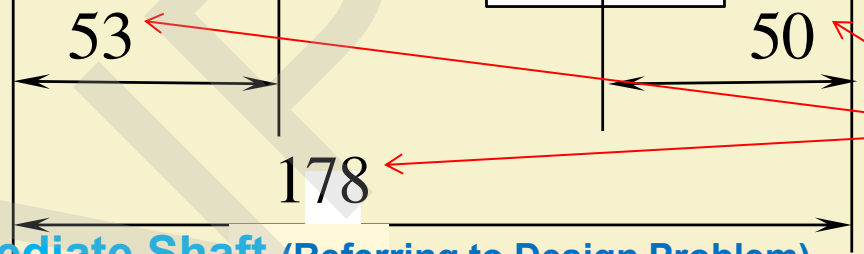
10 to 15 mm (Typical)

15 to 25 mm
(Typical)

(Not taper roller
in this case).

Considering
bearing widths,

finally distances
are marked.



Layout of Intermediate Shaft (Referring to Design Problem)

Dimensions of Gears and Gear data Table :

2nd. Step (Contd.):

Sl. No.	Description	First Stage		Second Stage	
		Pinion	Gear	Pinion	Gear
1.	Z , Number of Teeth	17	81	16	131
2.	Profile	20° Involute Full Depth, Un corrected			
3.	m_n , Normal module	3 mm		4 mm	
4.	β , Helix Angle	$11^\circ 26' 52''$		$11^\circ 26' 52''$	
		RH	LH	LH	RH
5.	Addendum Height (mm) $f_a \times m_n = 1.0 \times m_n$	3.0		4.0	
6.	Dedendum Height (mm) $f_d \times m_n = 1.25 \times m_n$	3.75		5.0	

Dimensions of Gears and Gear data Table (contd....) : 2nd. Step (Contd.):

Sl. No.	Description	First Stage		Second Stage	
		Pinion	Gear	Pinion	Gear
7.	d_p , Pitch Circle Diameter (PCD) (mm)	52.04	247.96	65.306	534.69
8.	d_a , Addendum or Tip Diameter (mm)	58.04	253.96	73.30	542.70
9.	d_d Dedendum or Root Diameter (mm)	44.54	240.46	55.30	524.70
10.	b , Face width. (mm)	63	58	68	63
11.	Material	EN 19A	EN 18A	EN 19A	EN 18A
12.	Surface Hardness (BHN) (Through Hardened)	350	300	350	300
p and g may be added to subscript of Nomenclature to indicate pinion and gear respectively. Similarly 1 and 2 can be added to indicate stage of Gear.					

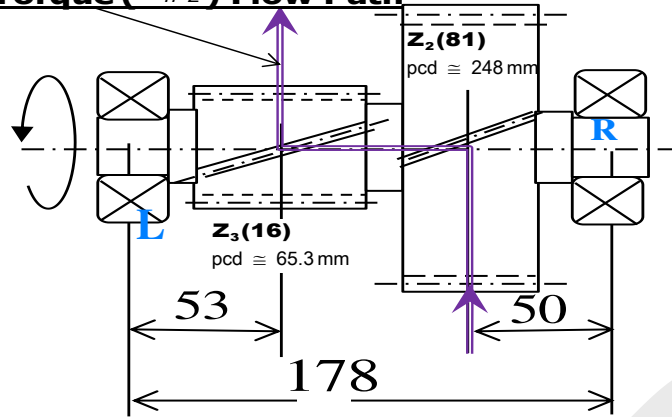
End of Step 2. Now the 1st. Layout of pinions and gears in mesh is done (Step 3) and Rough shape to the shafts are given (Step 4).

Bearing Life Estimation

Intermediate Shaft Bearings

5th. Step:

Torque (T_{n2}) Flow Path



Intermediate Shaft with gears and Bearings (Plan View)

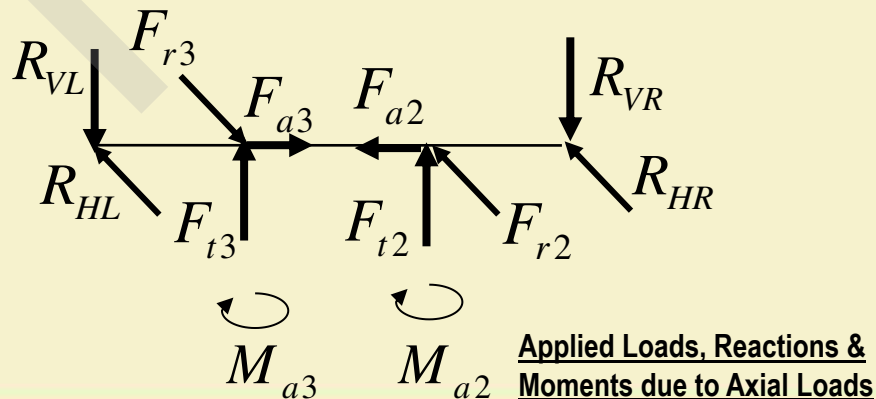
$$\begin{aligned}
 F_{r2} &= F_{t2} \sec \beta_1 \tan \alpha_n \\
 &= 1193.5 \times \sec(11^\circ 28' 42'') \times \tan 20^\circ \\
 &= 1193.5 \times 0.3714 = 443 \text{ N}
 \end{aligned}$$

Loads from Gear teeth are estimated as follows:

Tangential Load:

$$F_{t2} = \frac{2T_2}{d_{pg2}} = \frac{2 \times 31 \times (81/17)}{0.248} = 1193.5 \text{ N}$$

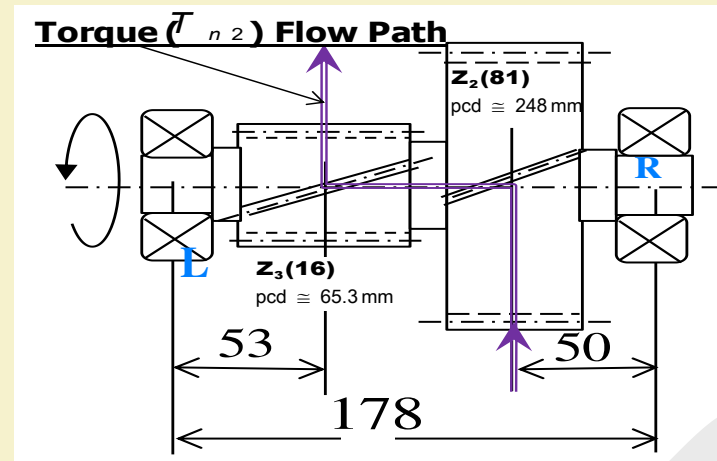
Note: Output nominal Torque of Motor=31.1 Nm (Approx)



Bearing Life Estimation

Intermediate Shaft Bearings

5th. Step:

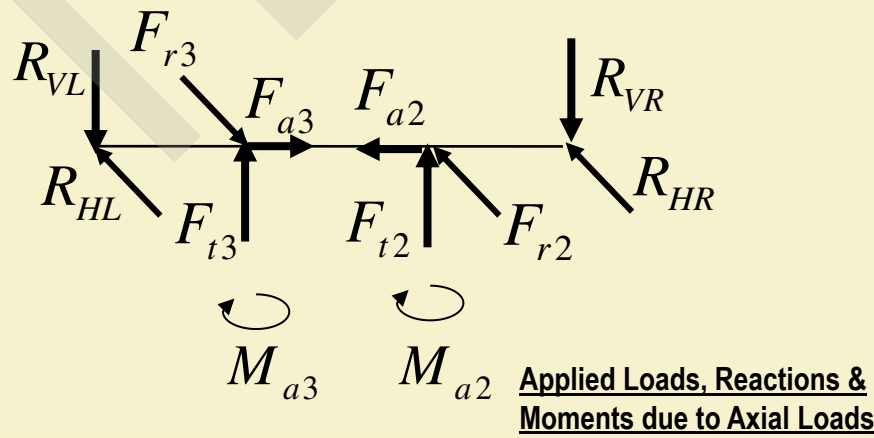


Intermediate Shaft with gears and Bearings (Plan View)

Loads from Gear teeth are estimated as follows:

Axial Load:

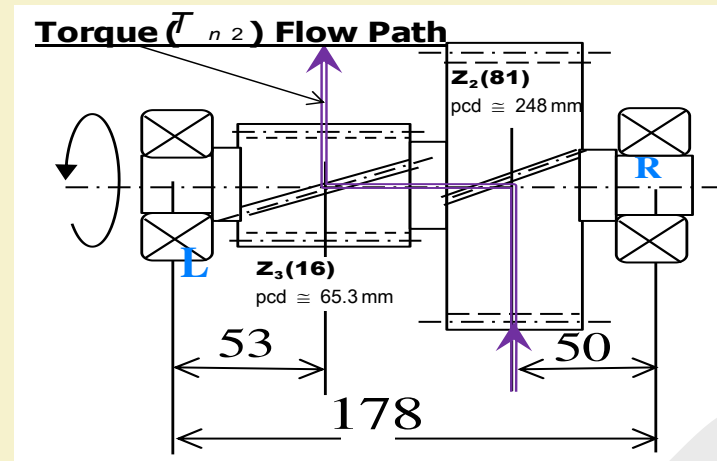
$$F_{a2} = F_{t2} \tan \beta_1 = 1193.5 \times \tan (11^\circ 28' 42'') \\ = 1193.5 \times 0.202 = 240.65 \text{ N}$$



Bearing Life Estimation

Intermediate Shaft Bearings

5th. Step:

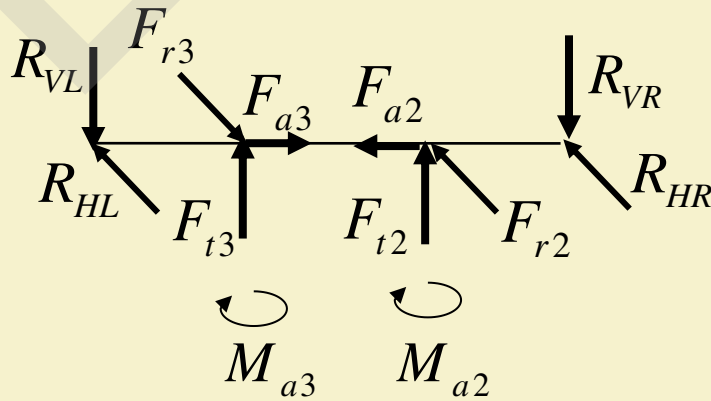


Intermediate Shaft with gears and Bearings (Plan View)

Loads from Gear teeth are estimated as follows:

Tangential Load:

$$F_{t2} = \frac{2T_2}{d_{pg2}} = \frac{2 \times 31 \times (81/17)}{0.248} = 1193.5 \text{ N}$$



Applied Loads, Reactions & Moments due to Axial Loads

Loads from 1st stage gear teeth on intermediate shaft are estimated as:

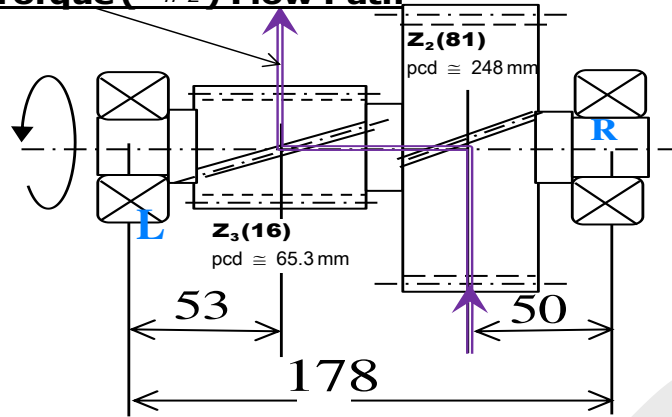
$$F_{t2} = 1193.5 \text{ N} \quad F_{r2} = 443 \text{ N} \quad F_{a2} = 240.65 \text{ N}$$

Bearing Life Estimation

Intermediate Shaft Bearings

5th. Step:

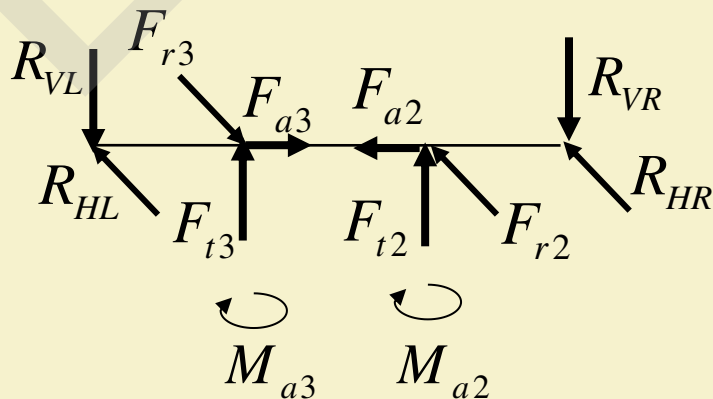
Torque (T_{n2}) Flow Path



Intermediate Shaft with gears and Bearings (Plan View)

Similarly, loads from 2nd stage pinion teeth on intermediate shaft are estimated as:

$$F_{t3} = \frac{2T_2}{d_{pp3}} = \frac{2 \times 31 \times (81/17)}{0.0653} = 4533 N$$



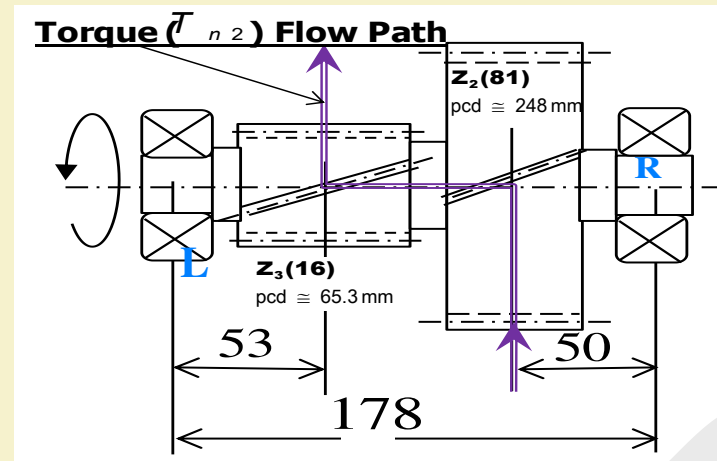
Applied Loads, Reactions & Moments due to Axial Loads

$$F_{t3} = 4533 N \quad F_{r3} = 1683 N \quad F_{a3} = 914 N$$

Bearing Life Estimation

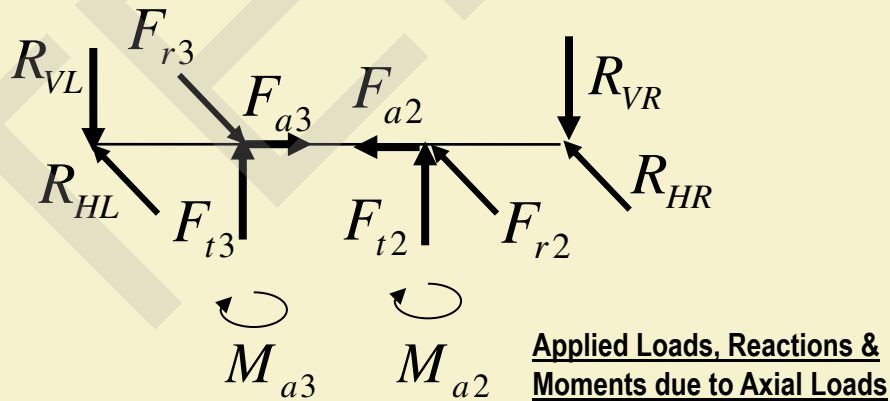
Intermediate Shaft Bearings

5th. Step:



Intermediate Shaft with gears and Bearings (Plan View)

Also, moments due to axial forces were estimated as: $M_{a2} = M_{a3} = 30 \text{ Nm}$



$R_{HL} = 720.6 \text{ N}$ $R_{VL} = 3518.5 \text{ N}$

$R_{HR} = 520 \text{ N}$ $R_{VR} = 2208 \text{ N}$

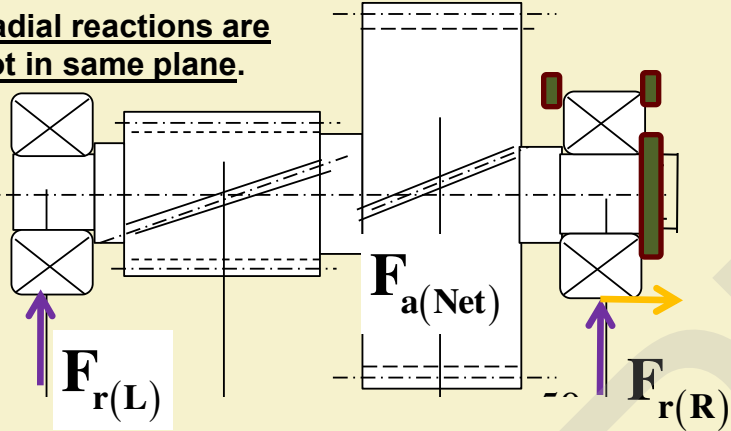
Finally Bearing reactions (radial) are estimated as:

Bearing reactions (axial) yet to be estimated.

Bearing Life Estimation 5th. Step (Contd...):

The Final bearing reactions:

Radial reactions are not in same plane.



Bearing Reactions (& Locking)

$F_{r(L)}$

11.57°

$F_{r(R)}$

From details of loading resultant right bearing (radial) reaction is calculated as:

$$F_{r(R)} = \sqrt{R_{VR}^2 + R_{HR}^2} = \sqrt{2208^2 + 520^2} \\ = 2268.4 \text{ N}$$

It is acting at an angle θ_R with vertical plane,

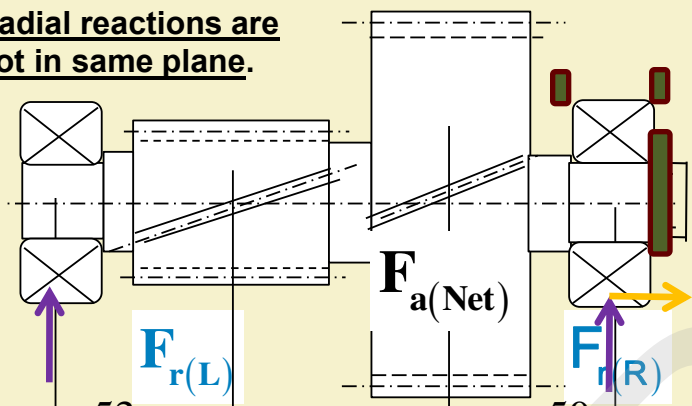
derived as $\theta_R = \tan^{-1}(R_{HR}/R_{VR}) = 13.25^\circ$

$$\text{Similarly, } F_{r(L)} = \sqrt{R_{VL}^2 + R_{HL}^2} = \sqrt{3518.5^2 + 720.6^2} \\ = 3591.5 \text{ N}$$

Bearing Life Estimation 5th. Step (Contd...):

The Final bearing reactions:

Radial reactions are not in same plane.

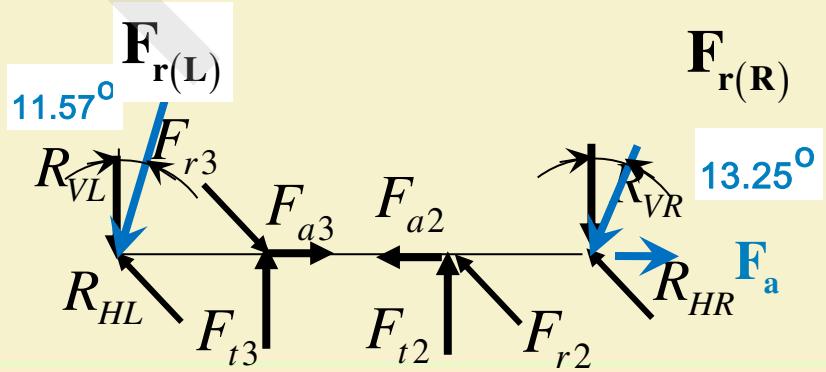


Bearing Reactions (& Locking)

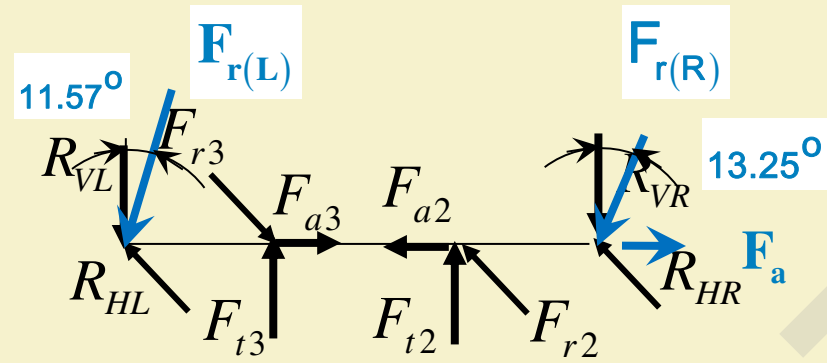
Resultant axial load may act only on one bearing irrespective of its direction (i.e., direction of shaft rotation).

$$F_{r(L)} = \sqrt{R_{VL}^2 + R_{HL}^2} = \sqrt{3518.5^2 + 720.6^2} = 3591.5 \text{ N}$$

and, $\theta_L = \tan^{-1}(R_{HL}/R_{VL}) = 11.57^\circ$



Bearing Life Estimation 5th. Step (Contd...):

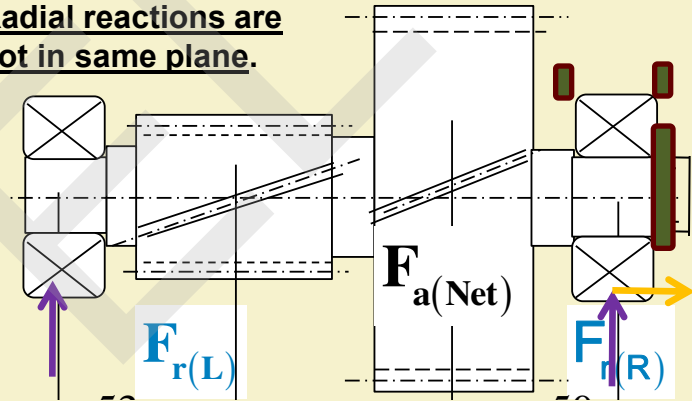


$F_{t3} = 4533\text{ N}$ $F_{r3} = 1683\text{ N}$ $F_{a3} = 914\text{ N}$
 $F_{t2} = 1193.5\text{ N}$ $F_{r2} = 443\text{ N}$ $F_{a2} = 240.65\text{ N}$
 $R_{HL} = 720.6\text{ N}$ $R_{VL} = 3518.5\text{ N}$
 $R_{HR} = 520\text{ N}$ $R_{VR} = 2208\text{ N}$

Details of loading & Resultant bearing Reactions.

The Final bearing reactions:

Radial reactions are not in same plane.



Bearing Reactions (& Locking)

It depends on bearing locking arrangement.

In this case it is on right bearing which is with less radial load.

Net axial load

$F_{a(Net)} = F_a = F_{a3} - F_{a2}$
 $= 673.35\text{ N}$

Thank You



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Module-04

Design of a General Purpose Industrial Helical Gear Reduction Unit- Part II

Lecture-18

Bearing Selection & Introduction to Shaft Design Verification

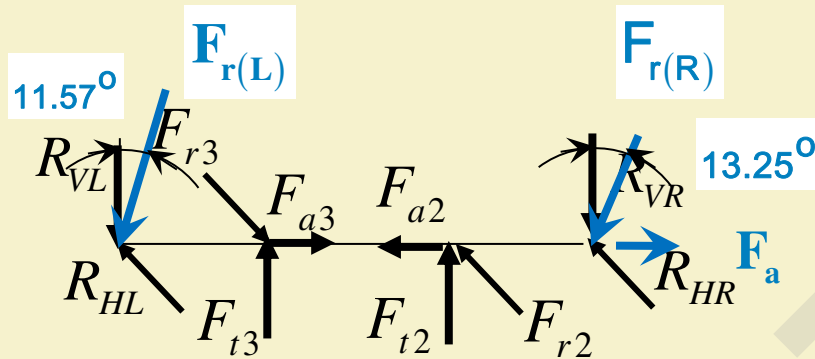


Outline of the Lecture

- **Bearing Selections for Intermediate Shaft**
 - **Choice of Bearing from Catalogue**
 - **Choice of Factors for Estimating Equivalent Reaction Loads on Bearing**
 - **Estimation of Equivalent Reaction Loads on Bearing**
 - **Estimation of Lives of Bearings**
 - **Search for more appropriate Bearings- Final Selection**
 - **Introduction to Design Verification of Shaft**
- **Calculation of Bending Moments on Shaft**



Loads on Shaft (Recapitulation)



$$F_{t3} = 4533 \text{ N} \quad F_{r3} = 1683 \text{ N} \quad F_{a3} = 914 \text{ N}$$

$$F_{t2} = 1193.5 \text{ N} \quad F_{r2} = 443 \text{ N} \quad F_{a2} = 240.65 \text{ N}$$

$$R_{HL} = 720.6 \text{ N} \quad R_{VL} = 3518.5 \text{ N}$$

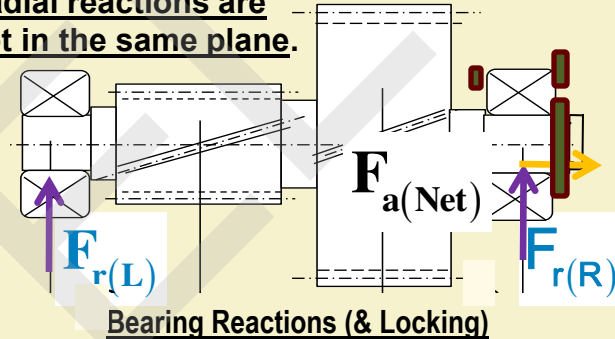
$$R_{HR} = 520 \text{ N} \quad R_{VR} = 2208 \text{ N}$$

Details of loading & Resultant bearing Reactions.

5th. Step (Contd...):

The Final bearing reactions:

Radial reactions are not in the same plane.



Resultant axial load may act only on one bearing irrespective of its direction (i.e., direction of shaft rotation).

Axial Load on Bearing depends on bearing locking arrangement.

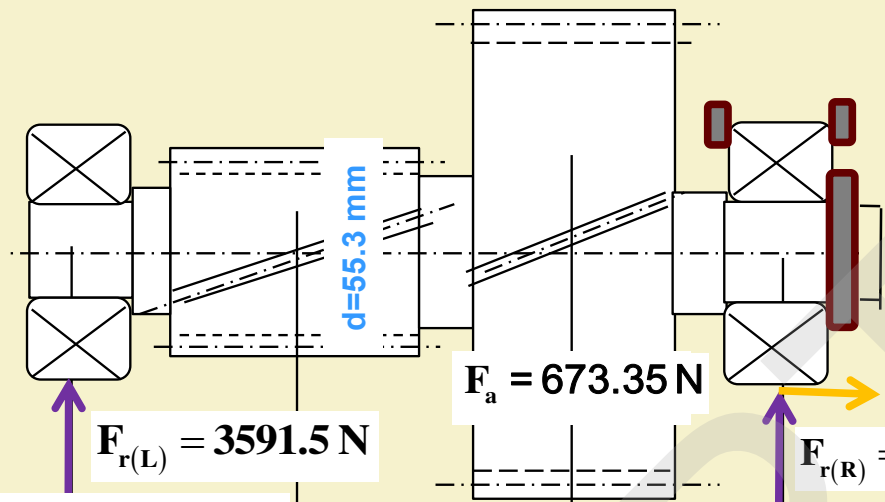
In this case it is on right bearing which is with less radial load.

Net axial load

$$F_{a(Net)} = F_a = F_{a3} - F_{a2} = 673.35 \text{ N}$$

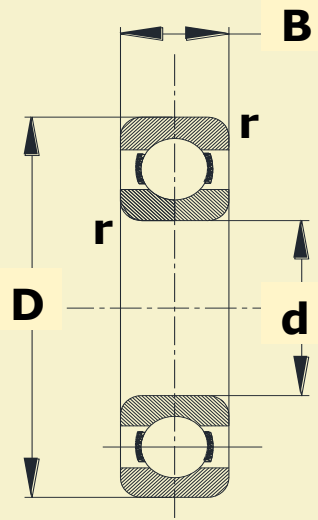
Bearing (Left) Life Estimation

5th. Step (Contd...):



Consider deep groove ball bearing **SKF 6309** as both end supports of intermediate shaft:

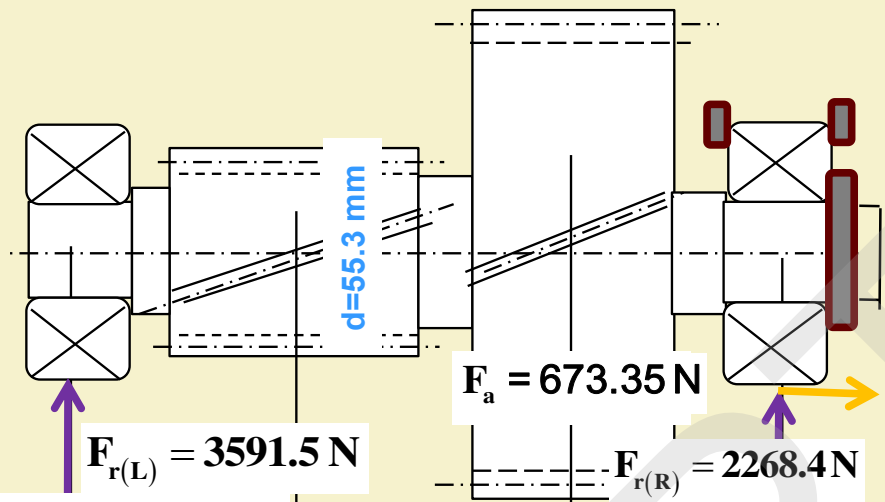
$X = 1 \text{ \& } Y = 1.6$



Bearing No.	Inner Dia. (d)	Outer Dia. (D)	Width (B)	Corner Radius (r) Approx.	Basic Load Capacity	
	mm	mm	mm	mm	Dynamic (C)	Static (C _o)
	mm	mm	mm	mm	Newton	Newton
6309	45	100	25	2.5	40130	29200

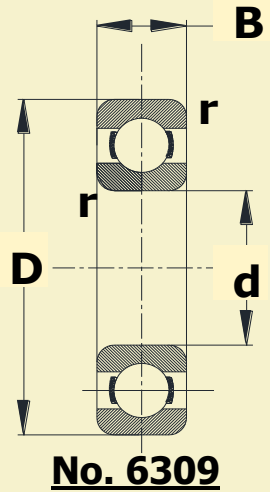
Bearing (Left) Life Estimation

5th. Step (Contd...):



Equivalent load on left bearing (No. 6309):

$$\begin{aligned}
 P_L &= C_1 \left(XVF_{r(L)} + YF_{a(L)} \right) \\
 &= 1.5 \times (1.0 \times 1 \times 3591.5 + Y \times 0) \\
 &= 5387.25 \text{ N}
 \end{aligned}$$

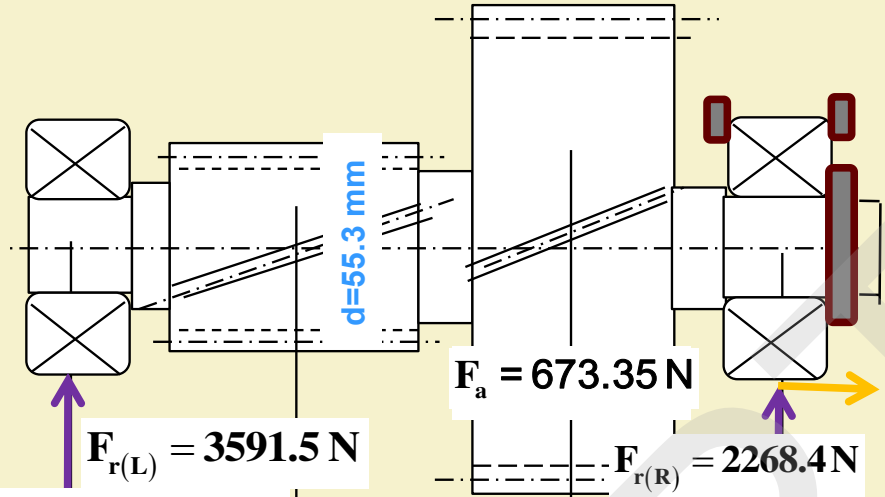


[Note: C_1 is taken as 1.5 considering medium shock load (given) on the estimated load on bearings based on nominal torque.]

Bearing No.	Inner Dia. (d)	Outer Dia. (D)	Width (B)	Corner Radius (r) Approx.	Basic Load Capacity	
	mm	mm	mm	mm	Dynamic C	Static C _o
6309	45	100	25	2.5	Newton	Newton
					40130	29200

Bearing (Left) Life Estimation

5th. Step (Contd...):



Life (in hrs) of left bearing:

$$L_{N(L)} = \left(\frac{C}{P} \right)^{\epsilon} \times 10^6 \text{ Revolution}$$

$$L_{H(L)} = \frac{L_{N(L)}}{N \times 60} = \frac{(40130 / 5387.5)^3 \times 10^6}{(1500 \times 17 / 81) \times 60}$$
$$= 0.021880 \times 10^6 \text{ hrs} = 21,880 \text{ hrs}$$

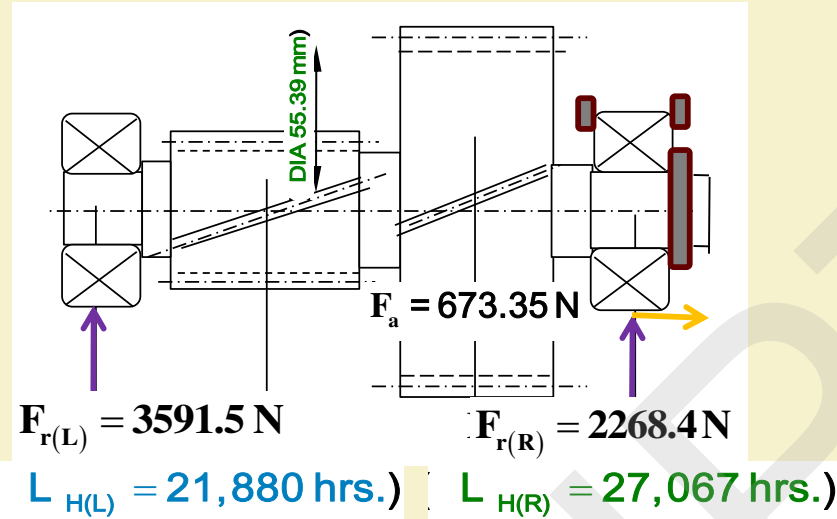
Bearing No.	Inner Dia. (d)	Outer Dia. (D)	Width (B)	Corner Radius (r) Approx.	Basic Load Capacity	
	mm	mm	mm	mm	Dynamic C	Static C _o
6309	45	100	25	2.5	40130	29200

It is more than double of required specified life (10,000 hrs).

Bearing (Right) Life Estimation

5th. Step (Contd...):

Ball bearing SKF 6309 is selected for both end supports of intermediate shaft:



Similarly, estimated equivalent load and life of right bearing:

$$P_R = 1.5 \times (1.0 \times 1 \times 2268.4 + 1.6 \times 673.35) = 5018.64 \text{ N,}$$

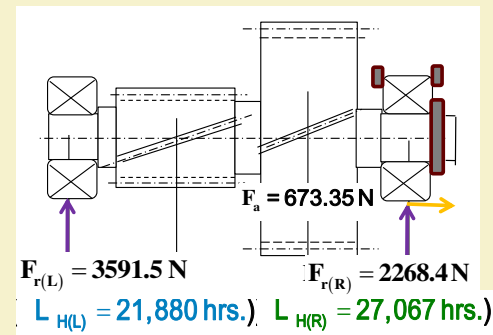
$$L_{H(R)} = \frac{(40130 / 5018.64)^3 \times 10^6}{(1500 \times 17 / 81) \times 60}$$

$$= 0.027067 \times 10^6 \text{ hrs} = 27,067 \text{ hrs}$$

Note: Estimated lives of both bearings are more or less same & above the required specified life (10,000 hrs).

Alternative Bearing (Right)- Life Estimation

Now it can be examined the life with bearing of lower load capacity:



Bearing No.	Inner Dia. (d) mm	Outer Dia. (D) mm	Width (B) mm	Corner Radius (r) Approx. mm	Basic Load Capacity	
					Dynamic (C) Newton	Static (C ₀) Newton
6309	45	100	25	2.5	40130	29200
6308	40	90	23	2.5	31000	21400
6211	55	100	21	2.5	32100	25415

Life in Revolution

$$L_{N(L)} = \left(\frac{C}{P} \right)^\epsilon \times 10^6$$

As the root diameter of pinion is 55.39 mm then a bearing of ID 55 mm (maximum) may be selected.

If SKF 6308 or 6211 is selected then life will **reduce by** $(C_{6309}/C_{6308 \text{ or } 6211})^3$ i.e., **2.17 or 1.95 times respectively**, which could be accepted.

However, design with SKF 6309 will perhaps be preferred.

End of Step- 5.

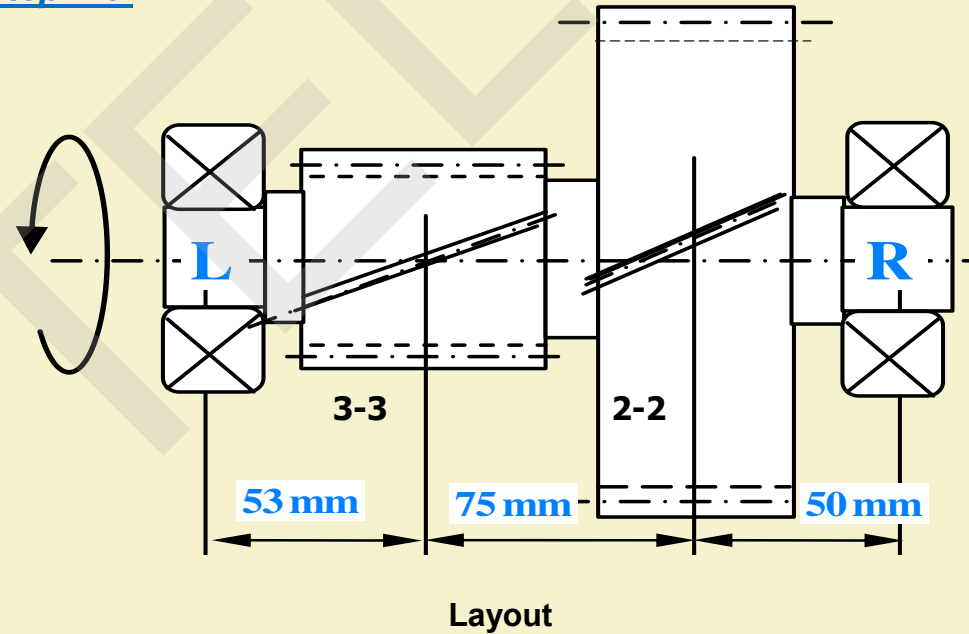
Design Verification of Intermediate Shaft

Bending moment on Intermediate Shaft Step – 6 due to Tangential Forces (Vertical Plane)

In case of gear box the diameters of a shaft is dominated by the size (root diameter) of the integral pinion and optimum bearing size mainly.

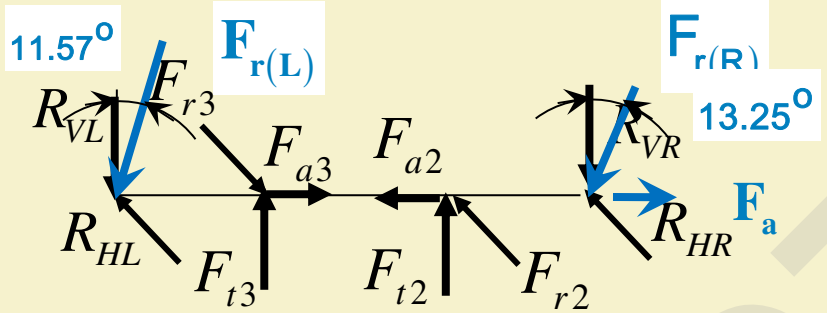
The length is determined by the placement of gear, pinion, bearings, coupling, key size, seals etc. and the optimum gap required between two consecutive elements.

Therefore, the shaft is automatically shaped during first layout and bearing selection, as discussed earlier.



Design Verification of Intermediate Shaft

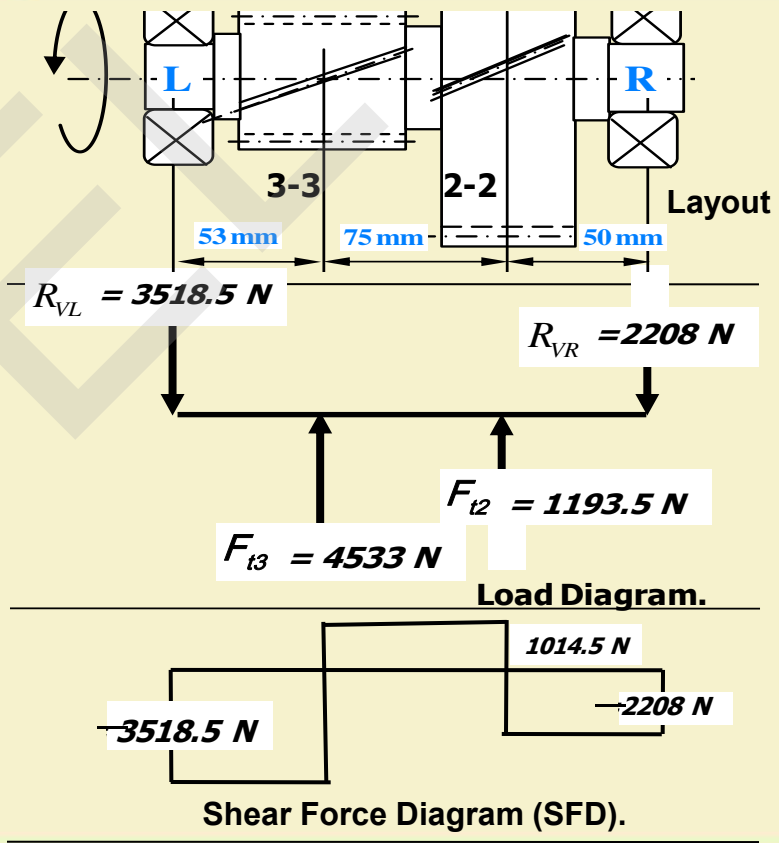
Bending moment on Intermediate Shaft Step – 6 due to Tangential Forces (Vertical Plane)



$F_{t3} = 4533 \text{ N}$
 $F_{t2} = 1193.5 \text{ N}$

$R_{VL} = 3518.5 \text{ N}$
 $R_{VR} = 2208 \text{ N}$

Details of loading & Resultant bearing Reactions.



Design Verification of Intermediate Shaft

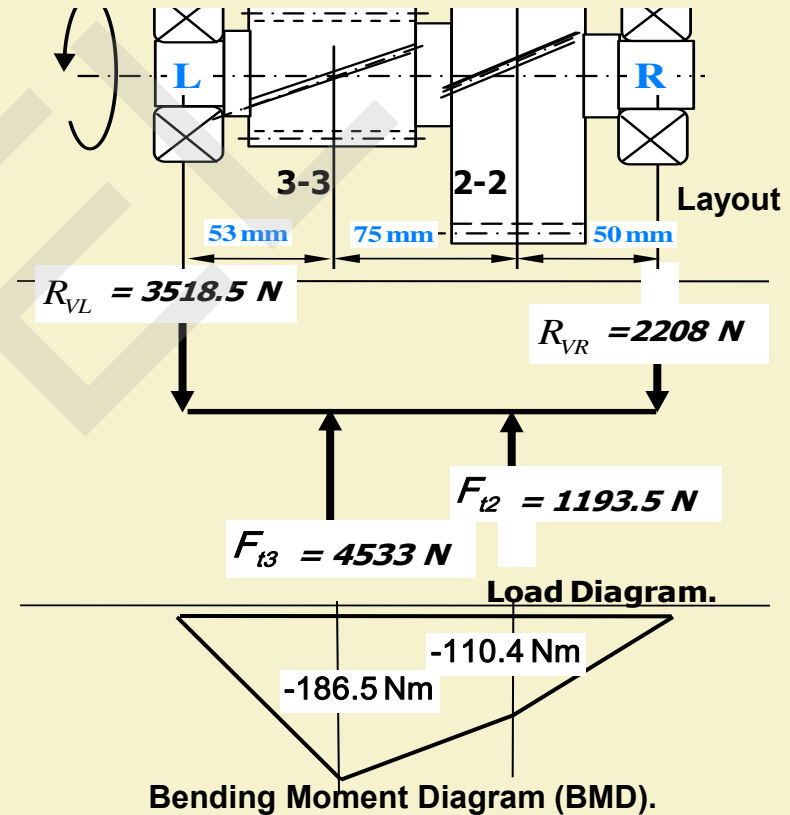
Bending moment on Intermediate Shaft Step – 6 due to Tangential Forces (Vertical Plane)

Therefore, instead of designing the shaft the critical sections are verified for developed stresses.

Bending Moment (respective plane) Calculation

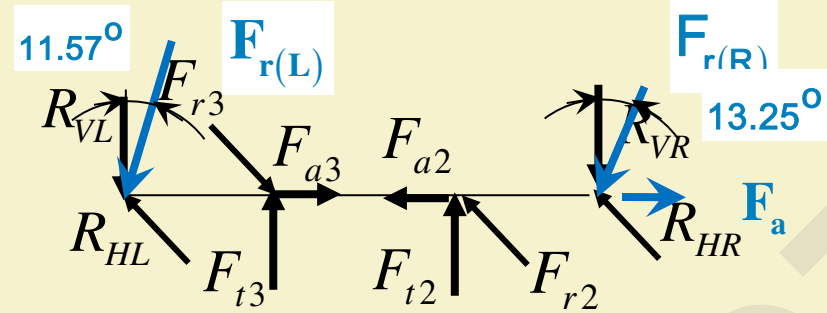
$$BM_{P3V} = -3518.5 \times 0.053 = -186.5 \text{ Nm}$$

$$BM_{G2V} = -3518.5 \times 0.128 + 4533 \times 0.075 = -110.4 \text{ Nm}$$



Design Verification of Intermediate Shaft

Bending moment on Intermediate Shaft Step – 6 due to Tangential Forces (Horizontal Plane)



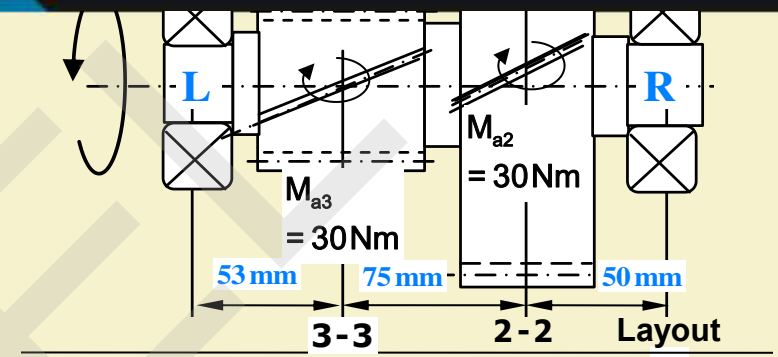
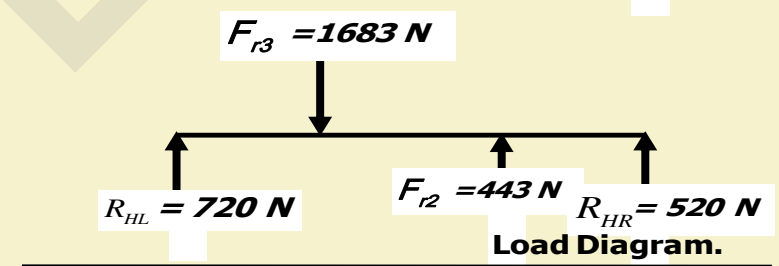
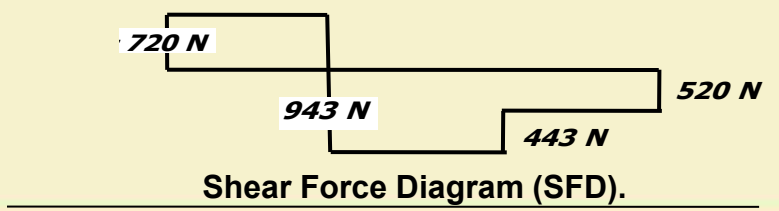
$F_{r3} = 1683\text{ N}$

$F_{r2} = 443\text{ N}$

$R_{HL} = 720.6\text{ N}$

$R_{HR} = 520\text{ N}$

Details of loading & Resultant bearing Reactions.



Design Verification of Intermediate Shaft

Bending moment on Intermediate Shaft Step – 6 due to Tangential Forces (Horizontal Plane)

Considering from left support Bending Moment just left of section 3-3:

$$BM_{P3H-} = 720 \times 0.053 = 38.2 \text{ Nm}$$

And just right of section 3-3:

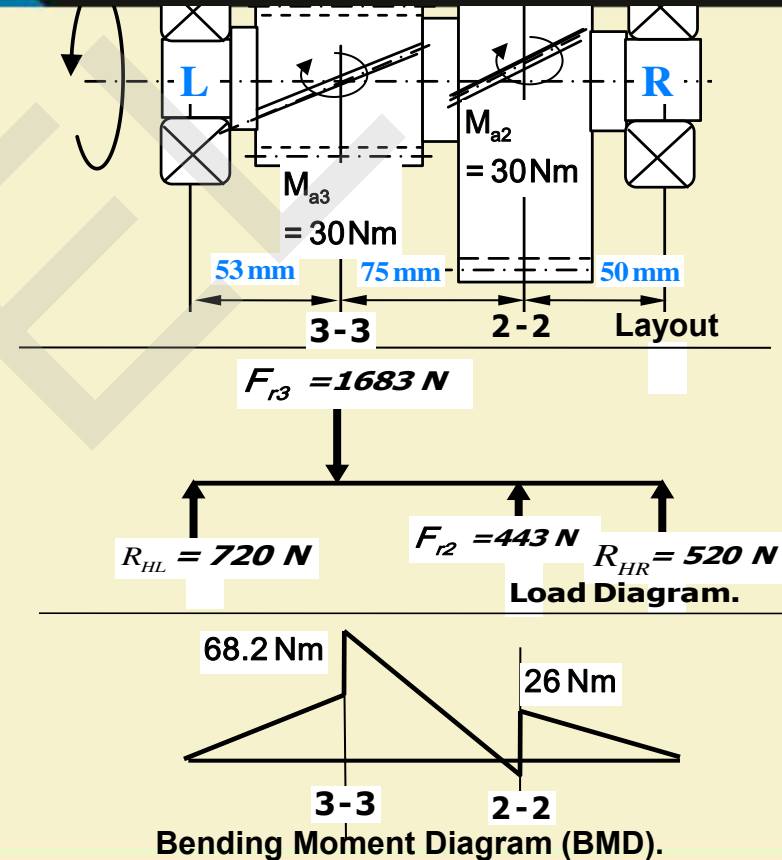
$$BM_{P3H+} = 38.2 + 30 = 68.2 \text{ Nm}$$

Similarly, BM just left of section 2-2:

$$BM_{G2H-} = 720 \times 0.128 - 1683 \times 0.075 + 30 = -4 \text{ Nm}$$

And BM just right of section 2-2:

$$BM_{G2H+} = -4.1 + 30 = 26 \text{ Nm}$$



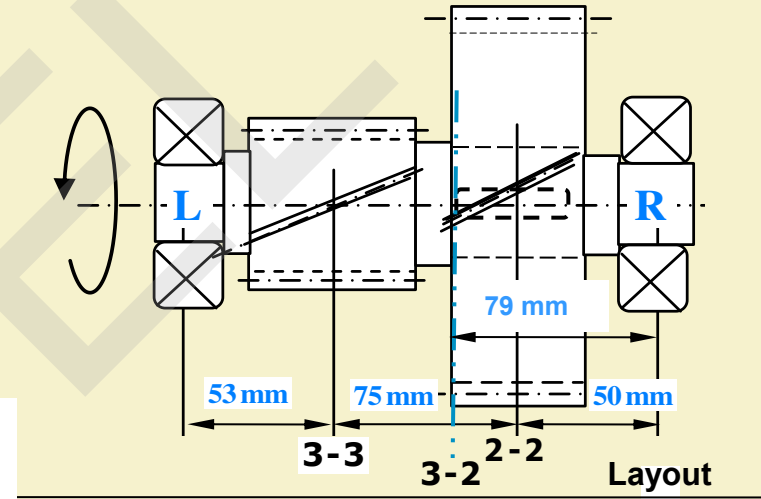
Design Verification of Intermediate Shaft

Step – 6

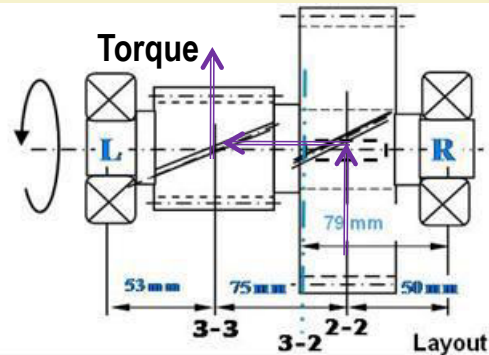
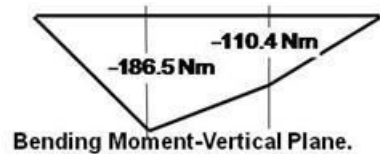
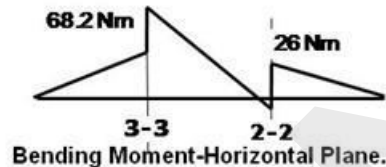
Second Step: Resultant Bending Moment and Critical Section

It is to be noted that in a rotating shaft outer layer experiences maximum flexural bending stress.

As bending stress is expressed by bending moment divided by section modulus, it is necessary to verify those for probable critical sections.



Resultant Bending Moment and Search for Critical Section (Contd...)



In the Intermediate shaft, any of sections 2-2, 3-2 & 3-3 may be critical i.e., experiences maximum bending stress.

Reasons are as follows:

- Among these three sections, through which full torque transmits, section 3-3 has maximum bending moment, although it has also the maximum diameter.
- It has medium stress concentration as it is roots of teeth.
- Sections 2-2 & 3-2 have equal diameters but different stress concentration factors.
- At section 3-2 there is step, where as at section 2-2 there is a keyway.
- Therefore, section 2-2 may be severe than section 3-2 in stress concentration point of view.
- Again section 2-2 usually experiences less Bending Moment.

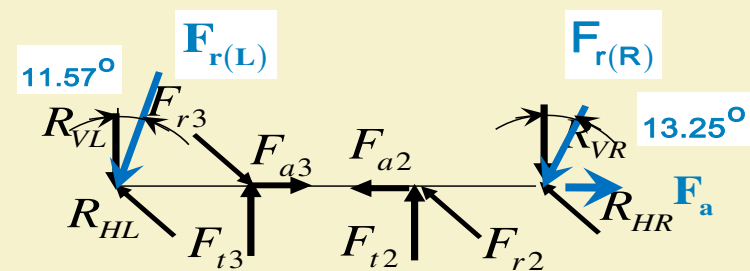
Design Verification of Intermediate Shaft

Step – 6 (Contd....)

Second Step: Resultant Bending

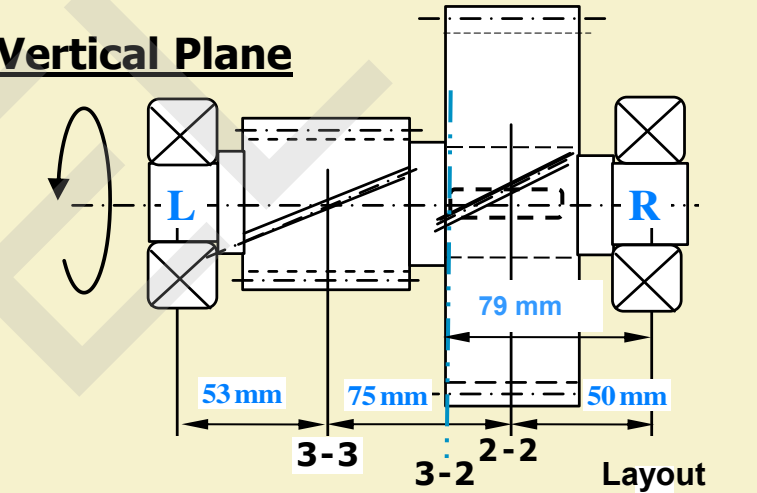
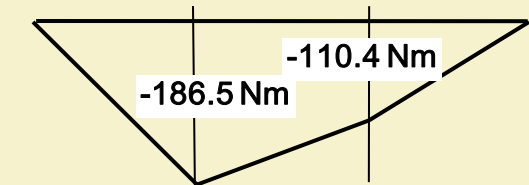
Moment and Critical Section

Bending Moment in Vertical Plane



$F_{t3} = 4533\text{ N}$ $F_{r3} = 1683\text{ N}$ $F_{a3} = 914\text{ N}$
 $F_{t2} = 1193.5\text{ N}$ $F_{r2} = 443\text{ N}$ $F_{a2} = 240.65\text{ N}$
 $R_{HL} = 720.6\text{ N}$ $R_{VL} = 3518.5\text{ N}$
 $R_{HR} = 520\text{ N}$ $R_{VR} = 2208\text{ N}$

Details of loading & Resultant bearing Reactions.

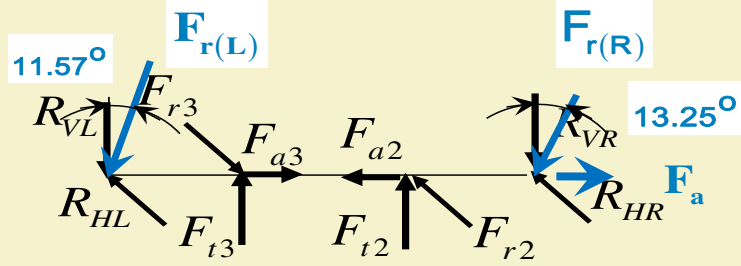


Bending Moment-Vertical Plane.

Second Step: Resultant Bending

Moment and Critical Section

Bending Moment in Horizontal Plane



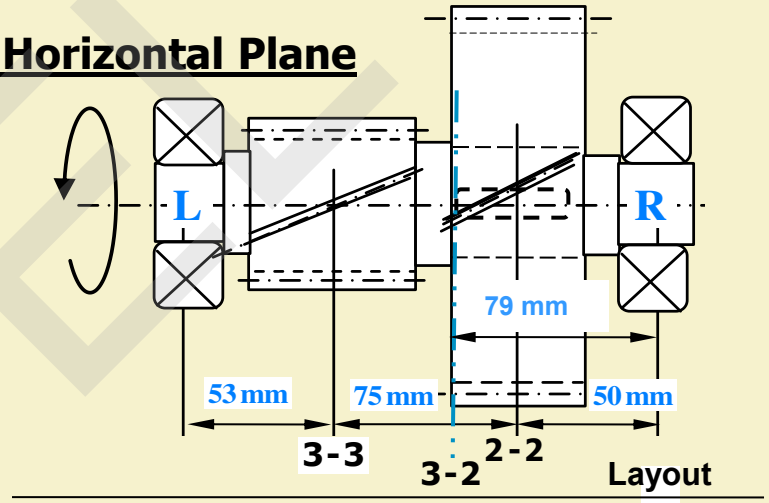
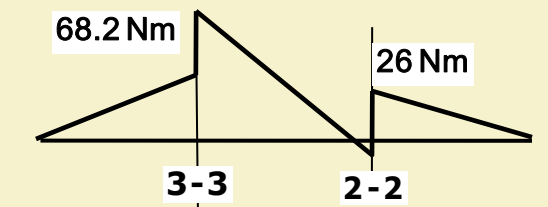
$F_{t3} = 4533\text{ N}$ $F_{r3} = 1683\text{ N}$ $F_{a3} = 914\text{ N}$

$F_{t2} = 1193.5\text{ N}$ $F_{r2} = 443\text{ N}$ $F_{a2} = 240.65\text{ N}$

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$R_{HR} = 520\text{ N}$ $R_{VR} = 2208\text{ N}$

Details of loading & Resultant bearing Reactions.



Bending Moment-Horizontal Plane.

Thank You



Module-04

Design of a General Purpose Industrial Helical Gear Reduction Unit- Part II

Lecture-19

Design Verification of Gear Box Shafts



Outline of the Lecture

- **Loads on (Intermediate Shaft) Shaft**
 - **Bending Moment at Vertical and Horizontal Planes**
 - **Search for Critical Sections**
 - **Resultant Bending Moments at Probable Weak Sections & Selection of Weakest Section**
 - **Formula for verifying Factor of Safety on Allowable Strength at critical section**
 - **Calculation of Different Stresses**
- **Verification of Factor of Safety (on Max Stresses in Intermediate shaft)**
 - **Hints on Design verification of Input and Output Shafts**

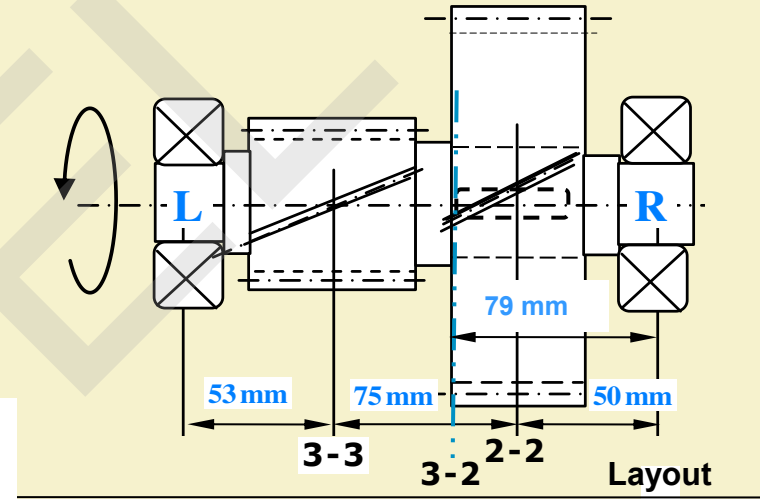
Design Verification of Intermediate Shaft

Step – 6

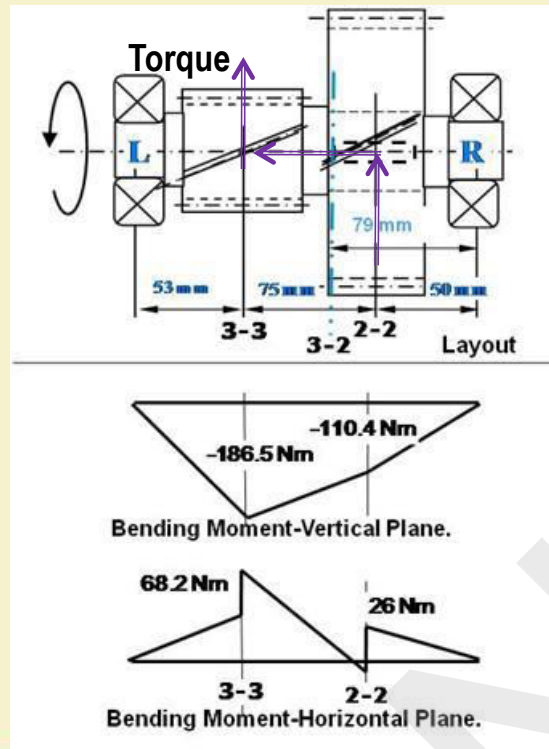
Second Step: Resultant Bending Moment and Critical Section

It is to be noted that in a rotating shaft outer layer experiences maximum flexural bending stress.

As bending stress is expressed by bending moment divided by section modulus, it is necessary to verify those for probable critical sections.



Resultant Bending Moment and Search for Critical Section (Contd...)



In the Intermediate shaft, any of sections 2-2, 3-2 & 3-3 may be critical i.e., experiences maximum bending stress.

Reasons are as follows:

- Among these three sections, through which full torque transmits, section 3-3 has maximum bending moment, although it has also the maximum diameter.
- It has medium stress concentration as it is roots of teeth.
- Sections 2-2 & 3-2 have equal diameters but different stress concentration factors.
- At section 3-2 there is step, where as at section 2-2 there is a keyway.
- Therefore, section 2-2 may be severe than section 3-2 in stress concentration point of view.
- Again section 2-2 usually experiences less Bending Moment.

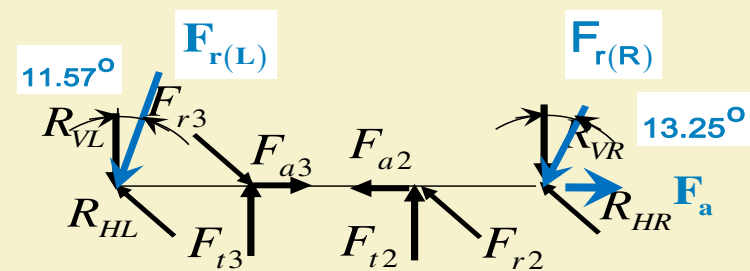
Design Verification of Intermediate Shaft

Step – 6 (Contd....)

Second Step: Resultant Bending

Moment and Critical Section

Bending Moment in Vertical Plane



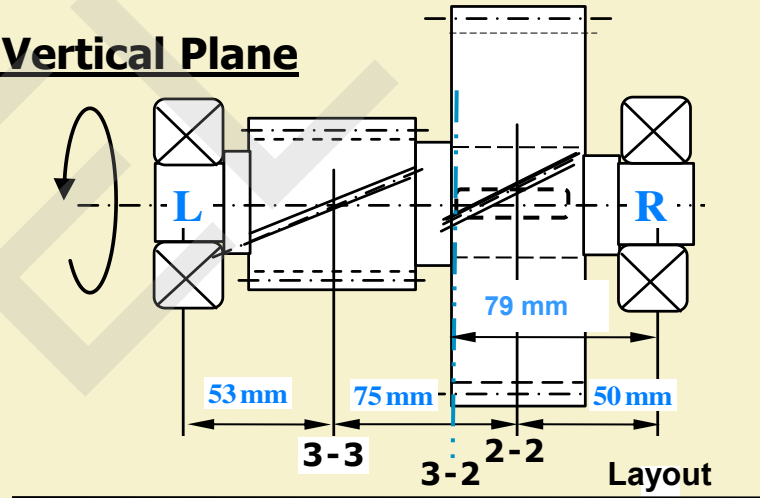
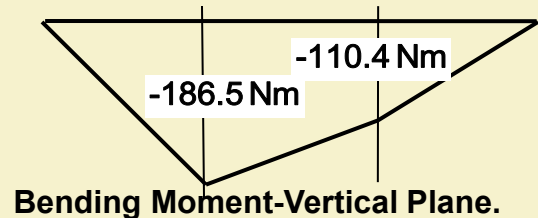
$F_{t3} = 4533 \text{ N}$

$F_{t2} = 1193.5 \text{ N}$

$R_{VL} = 3518.5 \text{ N}$

$R_{VR} = 2208 \text{ N}$

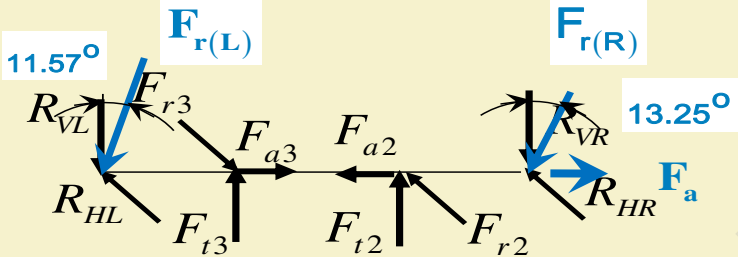
Details of loading & Resultant bearing Reactions.



Second Step: Resultant Bending

Moment and Critical Section

Bending Moment in Horizontal Plane



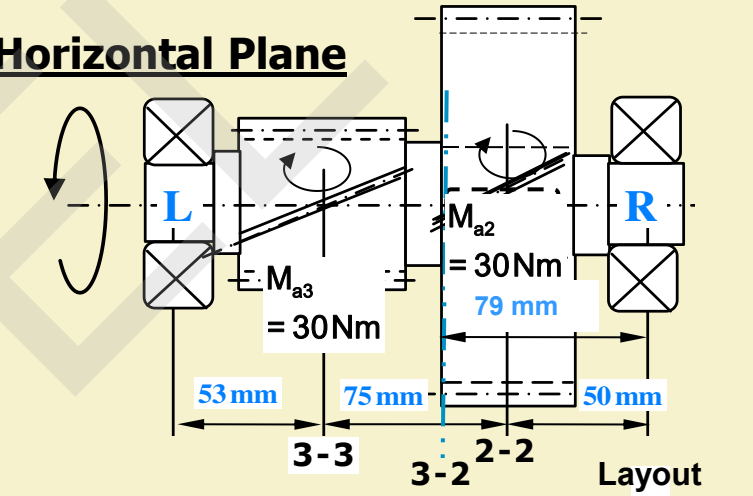
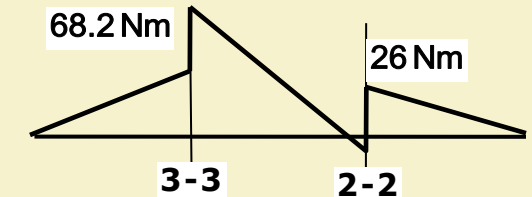
$F_{r3} = 1683 \text{ N}$

$F_{r2} = 443 \text{ N}$

$R_{HL} = 720.6 \text{ N}$

$R_{HR} = 520 \text{ N}$

Details of loading & Resultant bearing Reactions.



Design Verification of Intermediate Shaft

Step – 6 (Contd.....)

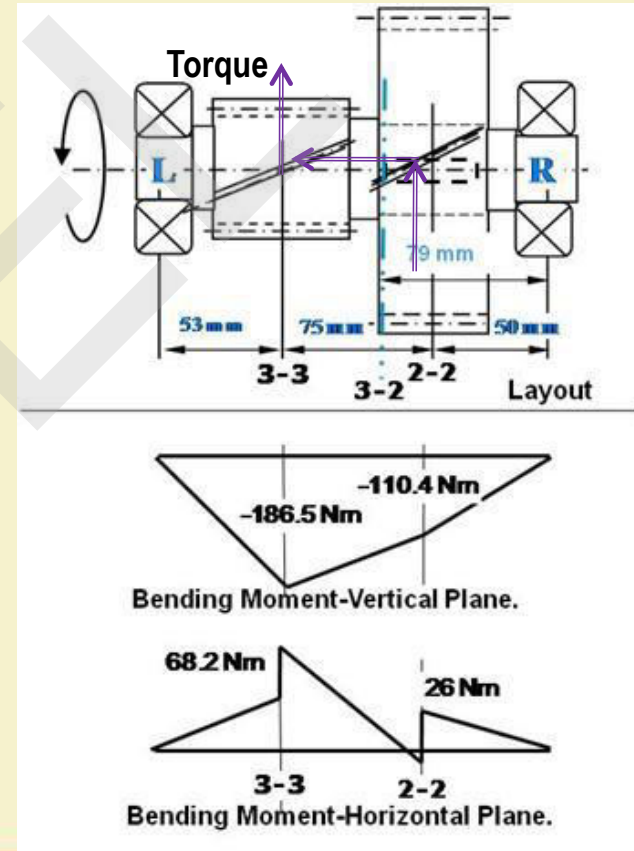
Resultant Bending Moment and Search for Critical Section (Contd....)

Resultant bending moment at 3-3:

$$BM_{R(3-3)} = \sqrt{68.2^2 + 186.5^2} = 198.6 \text{ Nm}$$

Resultant bending moment at 2-2:

$$BM_{R(2-2)} = \sqrt{26^2 + 110.4^2} = 113.42 \text{ Nm}$$



Design Verification of Intermediate Shaft

Step – 6 (Contd.....)

Resultant Bending Moment and Search for Critical Section (Contd....)

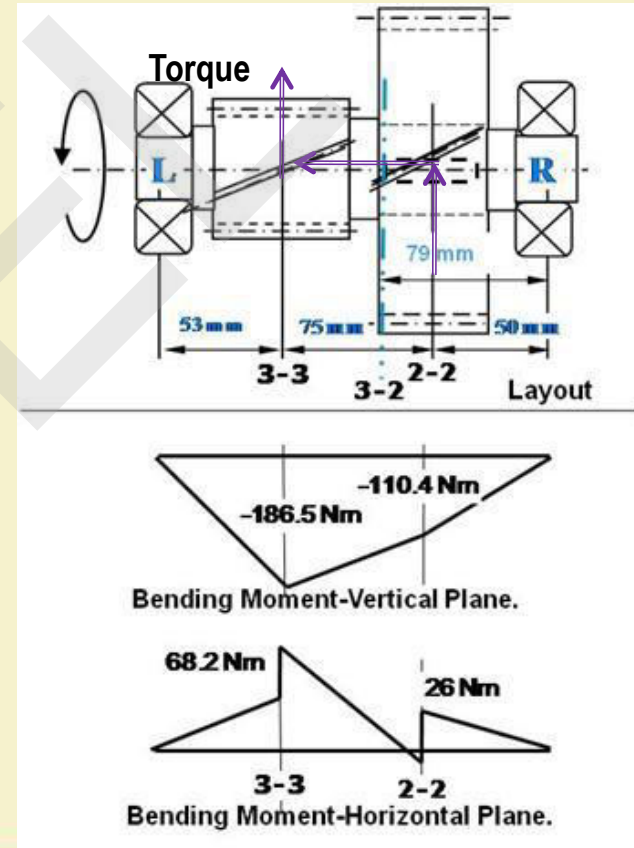
Resultant bending moment at 3-2 is estimated as follows:

Considering moment equilibrium about right support bearing:

$$BM_{V(3-2)} = 3518.5 \times 0.099 - 4533 \times 0.046 = 139.81 \text{ Nm}$$

$$BM_{H(3-2)} = 720.6 \times 0.099 + 30 - 1683 \times 0.046 = 23.92 \text{ Nm}$$

$$BM_{R(3-2)} = \sqrt{23.92^2 + 139.81^2} = 141.84 \text{ Nm}$$



Design Verification of Intermediate Shaft

Step – 6 (Contd.....)

Verification of Overall factor of safety at Critical Section (Contd.....)

Then a **factor of safety** f_s can be estimated using the following formula, which is base on **maximum shear stress theory under combined, bending, torsion and direct normal stresses.**

$$\frac{S_y}{f_s} = \sqrt{\left(\sigma_m + k_f \frac{S_y}{S_{en}} \sigma_a \right)^2 + 4\tau_m^2}$$

Where,

S_y = Yield strength of shaft material

S_{en} = Endurance strength of shaft material

σ_m = Mean (average) stress at considered section due to axial load.

σ_a = Maximum alternating stress at considered section due to bending.

τ_m = Maximum shear stress at considered section due to torsion.

k_f = A factor considering the feature of section and severity of service.

It is chosen considering on what basis σ_a has been calculated.

Design Verification of Intermediate Shaft

Step – 6 (Contd.....)

Bending Stresses and Search for Critical Section

Maximum bending stress in any section of rotating shaft (solid):

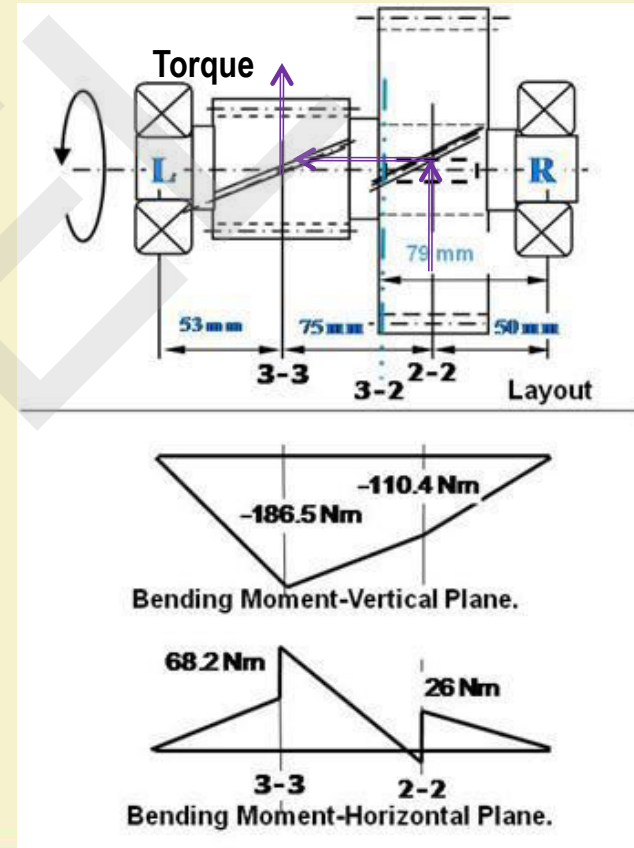
$$\sigma_b = f_c \frac{My}{I} = f_c \frac{32M}{\pi d^3}$$

(Section modulus $\frac{I}{y} = \frac{\pi d^4}{64}$ and

f_c stress concentration factor).

$$\sigma_{b(3-3)} = \frac{1.5 \times 32 \times 198.6}{\pi \times 0.0553^3} = 18 \times 10^6 \text{ Pas}$$

f_c is taken 1.5 for hob cut gear.



Design Verification of Intermediate Shaft

Step – 6 (Contd.....)

Bending Stresses and Search for Critical Section (Contd....)

Maximum bending stress at section 3-2:

$$\sigma_{b(3-2)} = \frac{1.5 \times 32 \times 141.84}{\pi \times 0.05^3} = 17.34 \times 10^6 \text{ Pas}$$

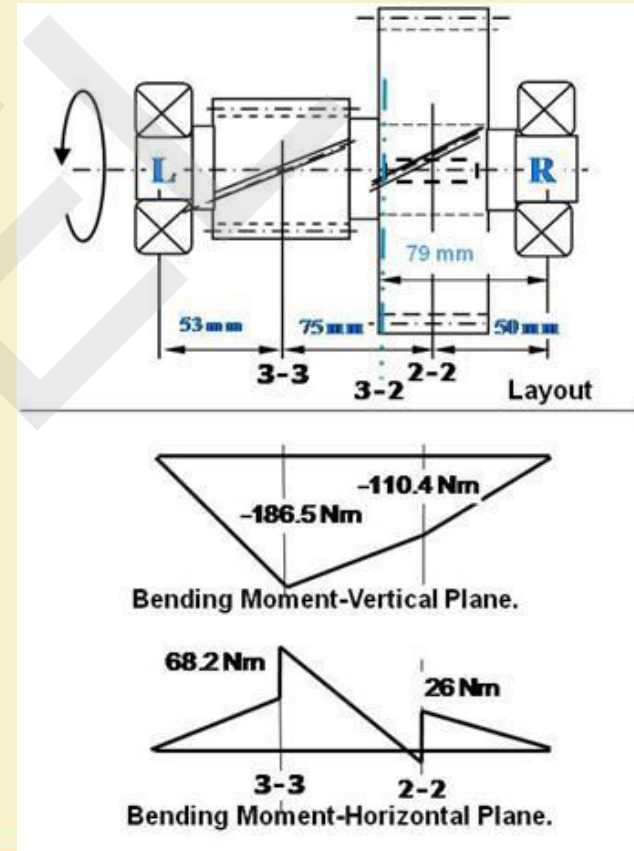
f_c is taken 1.5 for well designed step.

Maximum bending stress at section 2-2:

$$\sigma_{b(3-2)} = \frac{2 \times 32 \times 113.42}{\pi \times 0.05^3} = 18.5 \times 10^6 \text{ Pas}$$

f_c is taken 2 for milled keyway.

It is apparent that section 2-2 is critical.



Design Verification of Intermediate Shaft

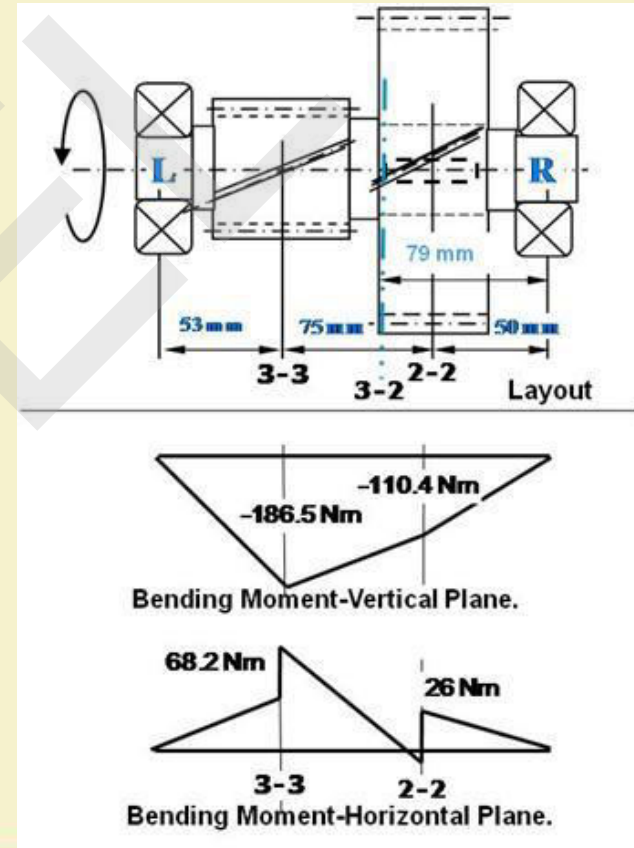
Step – 6 (Contd.....)

Lastly: Verification of Overall factor of safety at Critical Section

As already mentioned earlier, in gear unit design the size of the gear shaft is mostly biased by the sizes of gears, bearing layout and centre distances.

Particularly in case of shaft integral with the pinion there is little scope of pre-designing the shaft.

In such cases maximum stresses in the shaft are estimated identifying critical sections.



Design Verification of Intermediate Shaft

Step – 6 (Contd.....)

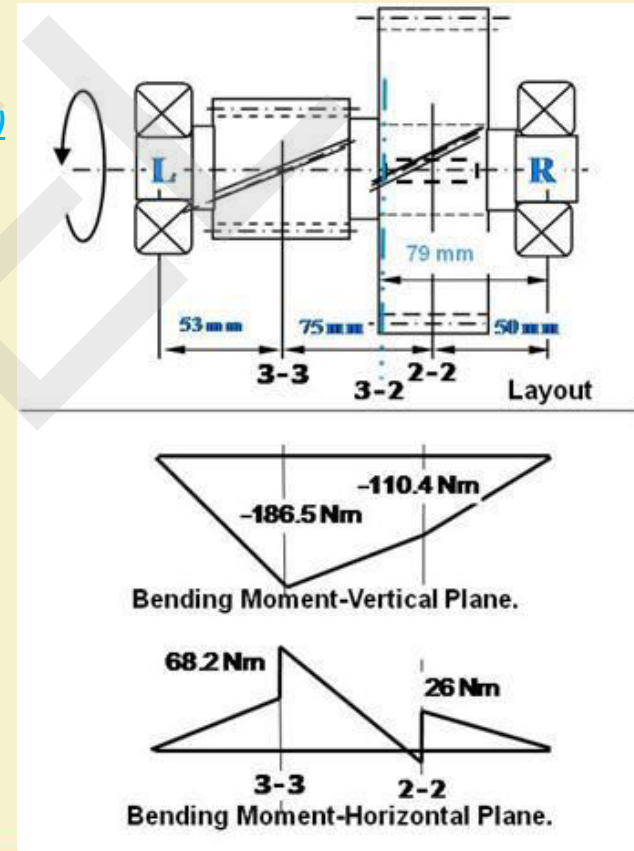
Verification of Overall factor of safety at Critical Section (Contd.....)

In present design, the pinion is integral with shaft therefore shaft material is EN19A.

Therefore, for the critical section 2-2:

$$S_y = 600 \text{ MPa},$$

$$S_{en} = 420 \text{ MPa (About 45\% of } S_u \text{ for well finished /ground shaft),}$$



Design Verification of Intermediate Shaft

Step – 6 (Contd.....)

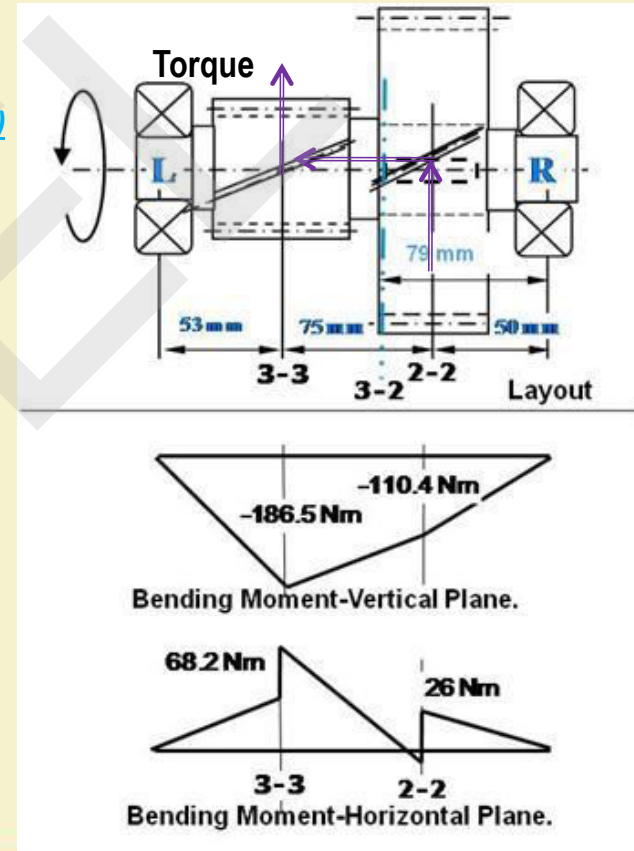
Verification of Overall factor of safety at Critical Section (Contd.....)

$$\sigma_m = f_c \frac{F_a}{\pi d^2} = 2 \times \frac{673.5}{\pi \times 0.05^2}$$
$$= 0.172 \times 10^6 \text{ Pas}$$

$$\sigma_a = \sigma_{b(2-2)} = 18.5 \times 10^6 \text{ Pas}$$

$$\tau_m = f_c \frac{16T}{\pi d^3} = 2 \times \frac{16 \times 148}{\pi \times 0.05^3}$$
$$= 12.1 \times 10^6 \text{ Pas}$$

f_c is taken 2 in general for milled single keyway.



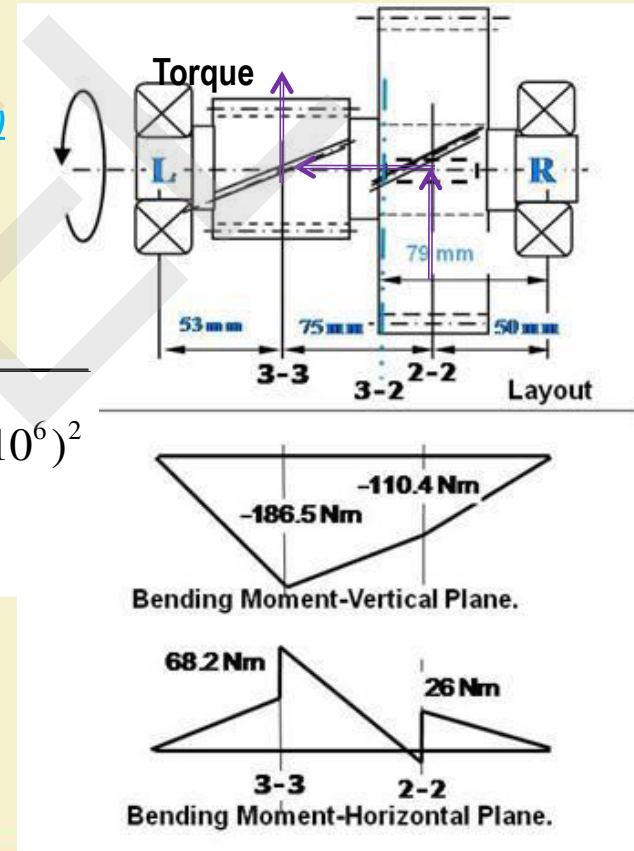
Design Verification of Intermediate Shaft

Step – 6 (Contd.....)

Verification of Overall factor of safety at Critical Section (Contd.....)

Substituting values f_s for the critical section 2-2 is calculated as follows:

$$\frac{600 \times 10^6}{f_s} = \sqrt{\left[\left(0.172 + 1.5 \times \frac{600}{420} \times 18.5 \right) \times 10^6 \right]^2 + 4 \times (12.1 \times 10^6)^2}$$
$$= 46.6 \times 10^6$$



Design Verification of Intermediate Shaft

Step – 6 (Contd.....)

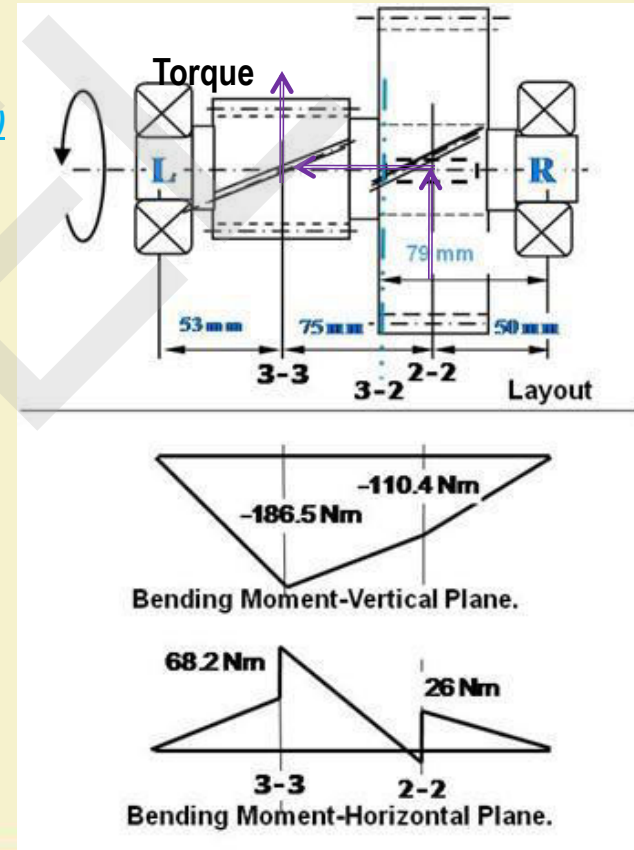
Verification of Overall factor of safety at Critical Section (Contd.....)

Therefore,

$$f_s = \frac{600}{46.6} = 12.87$$

This is highly satisfactory.

Usually f_s is taken as 2.5 to 3.



Design Verification of Input & Output Shaft

Step – 6 (Contd.....)

Input Shaft

The Input Shaft is also integral with the 1st. stage pinion.

Therefore, the material is EN19A.

Shaft design verification is done in the same way as it is done for intermediate shaft.

Output Shaft

The Output Shaft not integral with the gear.

Therefore, medium carbon steel (C40 or C45, Equivalent to EN8), having ultimate strength- 560 MPa and yield strength- 280 Mpa, is taken as the material.

The Shaft diameter is initially estimated on transmitted torque as follows:

Design Verification of Input & Output Shaft (Contd.....)

Step – 6 (Contd.....)

Output Shaft

The Shaft diameter is initially estimated on transmitted torque as follows:

$$d_o = \sqrt[3]{\frac{16T_o}{\pi S_{sa}}}$$

In the present design considering a factor of 1.5 with nominal torque the design torque for output shaft is taken as:

$$T_o = 1.5 \times 31 \times 39.1 = 1818 \text{ Nm}$$

Considering allowable shear stress (S_{sa}) of the selected material is 60 MPa.

$$\text{Nominal diameter } d_o = 53.65 \text{ mm}$$

Considering the end bearings of ID 55 mm (Say SKF Ball Bearing 6311) Shaft design verification is done in the same way as is done for intermediate shaft.

Thank You



Module-04

Design of a General Purpose Industrial Helical Gear Reduction Unit- Part II

Lecture-20

Development (Layout) of Intermediate Shaft



Outline of the Lecture

- **Design Problem, Gear Data & Dimensions & 1st. Layout (Recapitulation)**
 - **Refining the Drawing of Intermediate Shaft with followings**
 - **Development of Gear End Shaft & Bearing Placement**
- **Gear End Bearing Locking Arrangement- use of Standard Lock Washer & Nut**
 - **Development of Pinion End Shaft & Bearing Placement**
- **Pinion End Bearing Locking Arrangement- use of Circlip**
 - **Probable Bearing Locking Arrangement with housing.**

Dimensions of Gears and Gear data Table : (Recapitulation):

The Gear Unit design problem is same as already considered. All data remain same except the Second stage reduction ratio is reduced to 5.125. Overall transmission ratio is now 24.42.

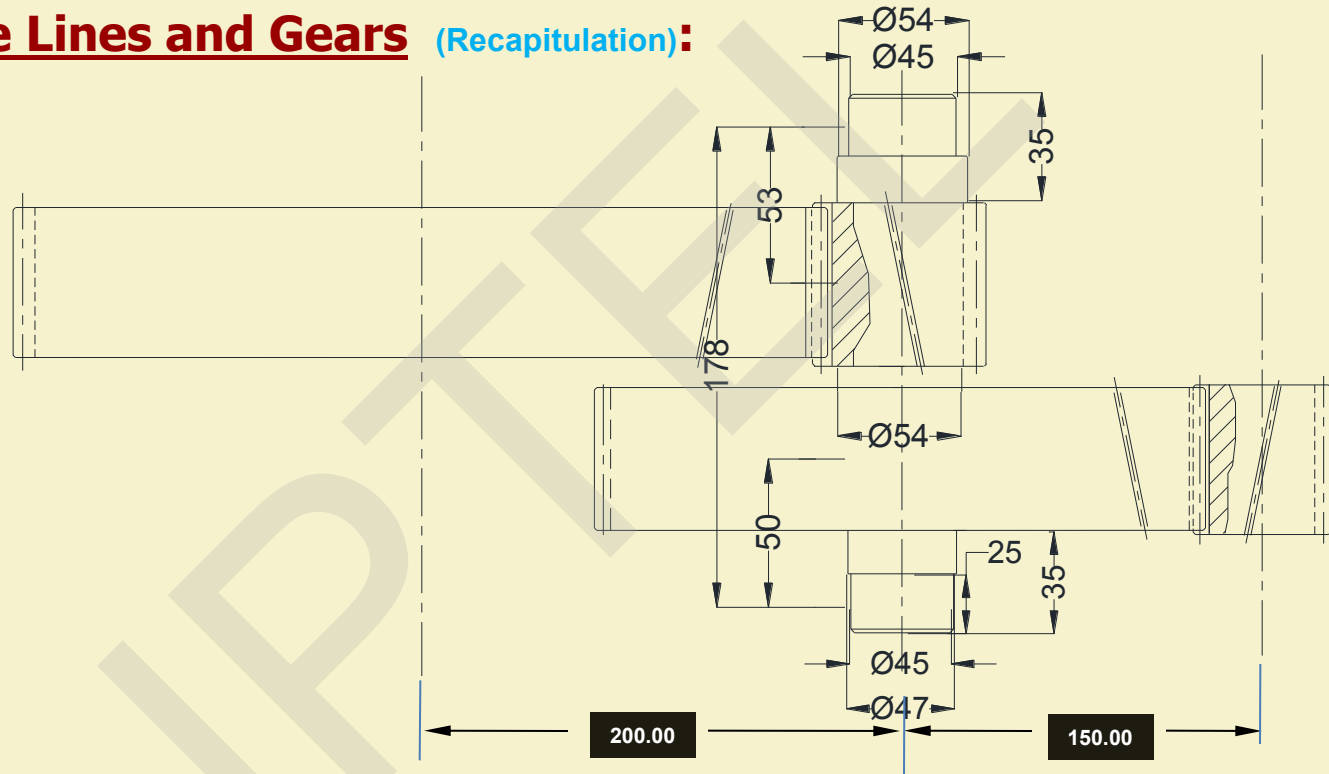
Sl. No.	Description	1 st . Stage		2 nd . Stage	
		Pinion	Gear	Pinion	Gear
1.	Number of Teeth (Z)	17	81	16	82
2.	Tooth Profile	20° Full Depth Involute, Uncorrected			
3.	Normal Module (m_n)	3 mm		4 mm	
4.	Helix Angle (β) and Direction of Helix	11°28'42"		11°28'42"	
		RH	LH	LH	RH
5.	Addendum Height, ($a_f \times m_n = 1 \times m_n$)	3 mm		4 mm	
6.	Dedendum Height, ($d_f \times m_n = 1.25 \times m_n$)	3.75 mm		5 mm	

Dimensions of Gears and Gear data Table : (Recapitulation):

Sl. No.	Description	1 st . Stage		2 nd . Stage	
		Pinion	Gear	Pinion	Gear
7.	Pitch Circle Diameter (mm)	52.04	247.96	65.306	334.694
8.	Centre Distance (mm)	150		200	
9.	Addendum or Tip Circle Diameter (mm)	58.04	253.96	73.306	342.694
10.	Dedendum or Root Circle Diameter (mm)	44.54	240.46	55.306	324.694
11.	Face width (b)	63 mm	58 mm	68 mm	63 mm
12.	Material	EN 19A	EN 18A	EN 19A	EN 18A
13.	Hardness , Through Hardened (BHN)	350	300	350	300

(Recapitulation)

2nd. Stage Pinion End

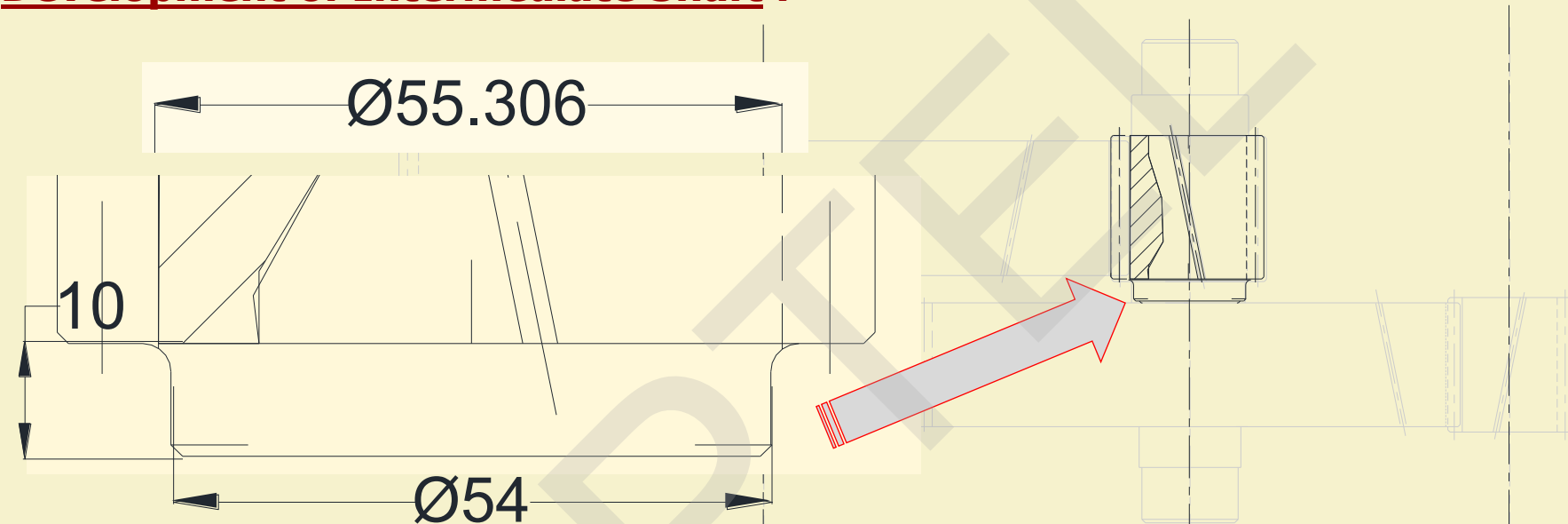


Sl. No.	Description	1 st . Stage		2 nd . Stage	
		Pinion	Gear	Pinion	Gear
10.	Dedendum or Root Circle Diameter (mm)	44.54	240.46	55.306	324.694

ALL DIMENSIONS ARE IN MM
EXCEPT OTHERWISE MENTIONED

Development of Intermediate Shaft :

Details Between Pinion and Gear :

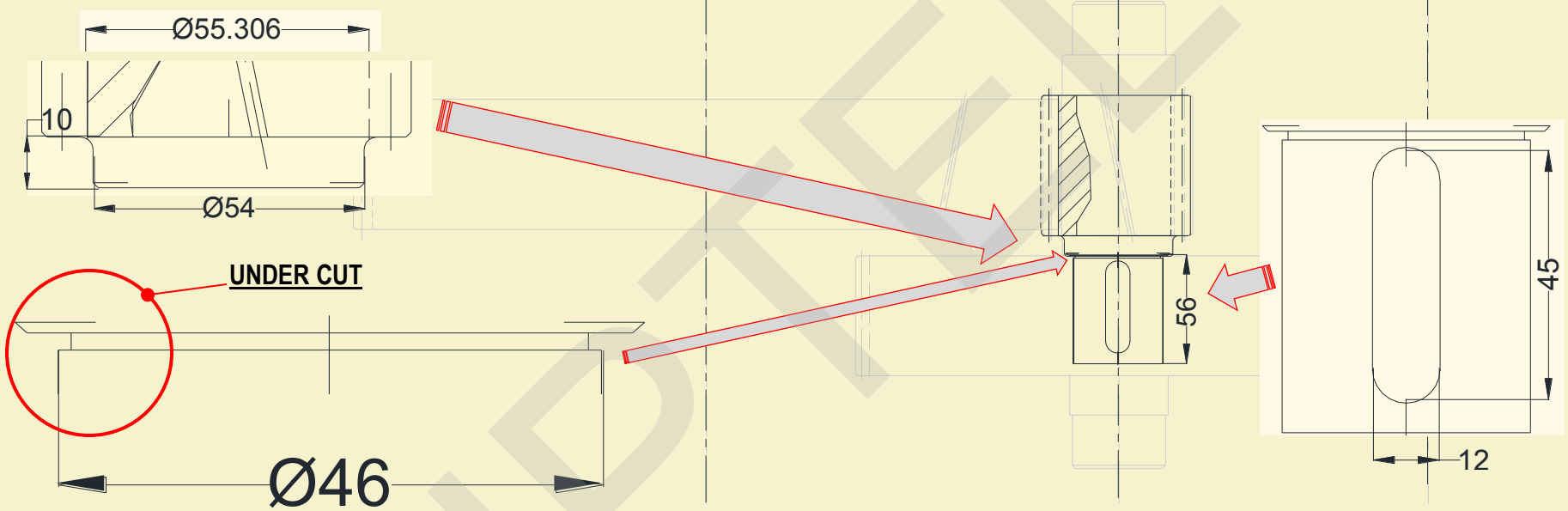


Sl. No.	Description	1 st . Stage		2 nd . Stage	
		Pinion	Gear	Pinion	Gear
10.	Dedendum or Root Circle Diameter (mm)	44.54	240.46	55.306	324.694

ALL DIMENSIONS ARE IN MM
EXCEPT OTHERWISE MENTIONED

Development of Intermediate Shaft (Contd...) :

Details for Mounting Gear :

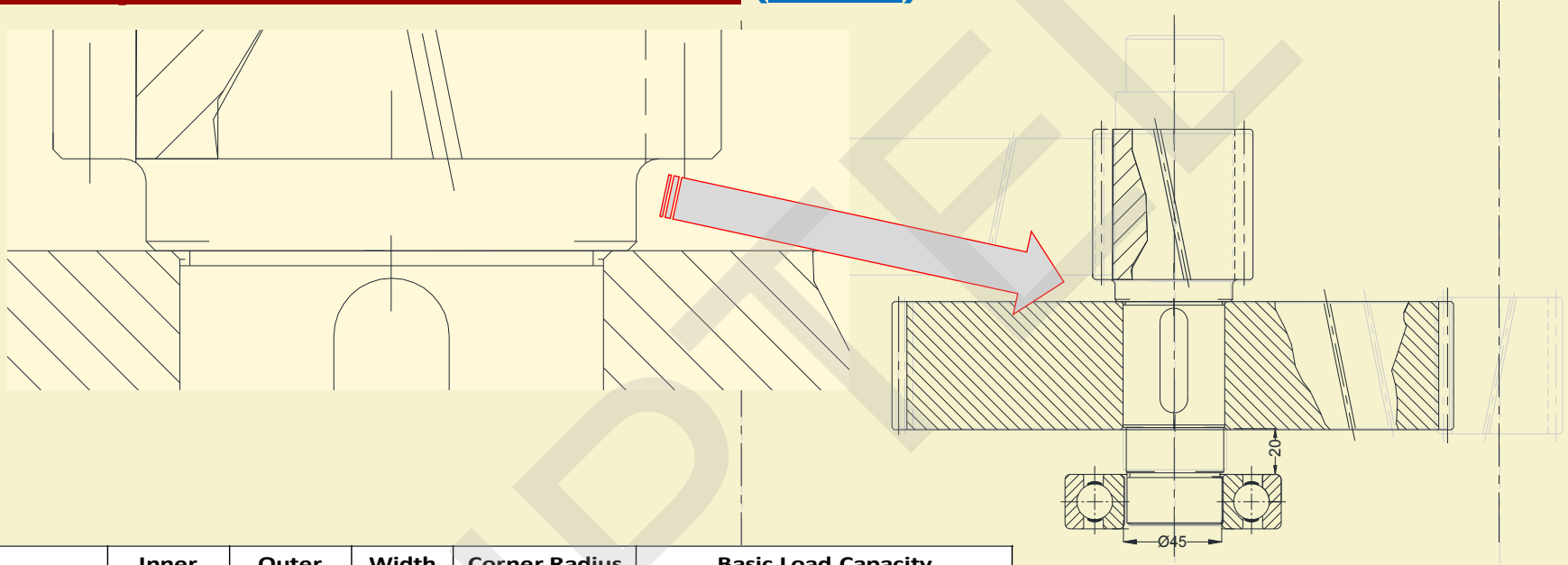


Sl. No.	Description	1 st . Stage		2 nd . Stage	
		Pinion	Gear	Pinion	Gear
11.	Face width (b)	63 mm	58 mm	68 mm	63 mm

ALL DIMENSIONS ARE IN MM
EXCEPT OTHERWISE MENTIONED

Development of Intermediate Shaft (Contd...) :

Details for Gear End Bearing :



Bearing No.	Inner Dia. (d)	Outer Dia. (D)	Width (B)	Corner Radius (r) Approx.	Basic Load Capacity	
	mm	mm	mm	mm	Dynamic (C)	Static (C ₀)
6309	45	100	25	2.5	40130	29200

ALL DIMENSIONS ARE IN MM
EXCEPT OTHERWISE MENTIONED

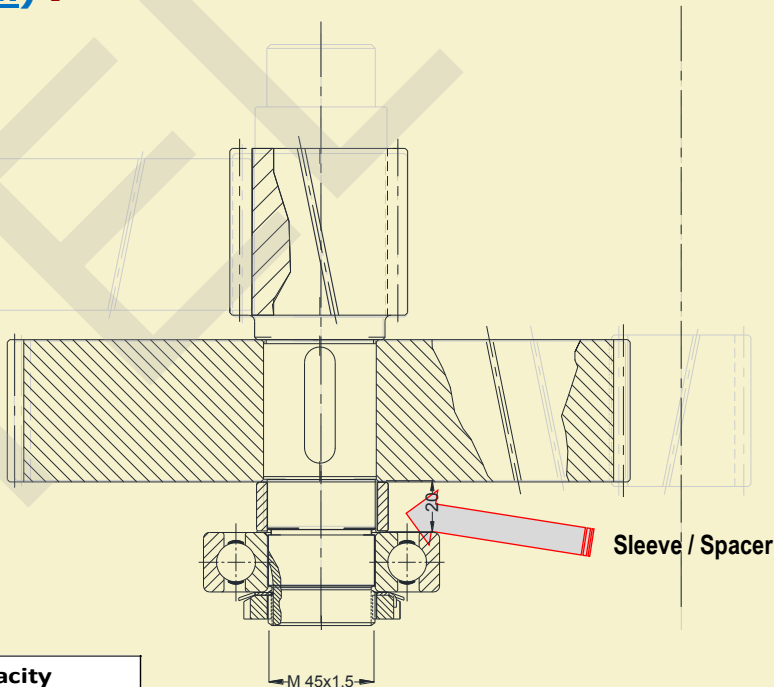
Development of Intermediate Shaft (Contd...) : **Gear End Bearing Locking with Shaft:**



Lock WASHER (MB 9)



Lock NUT (KM 9)

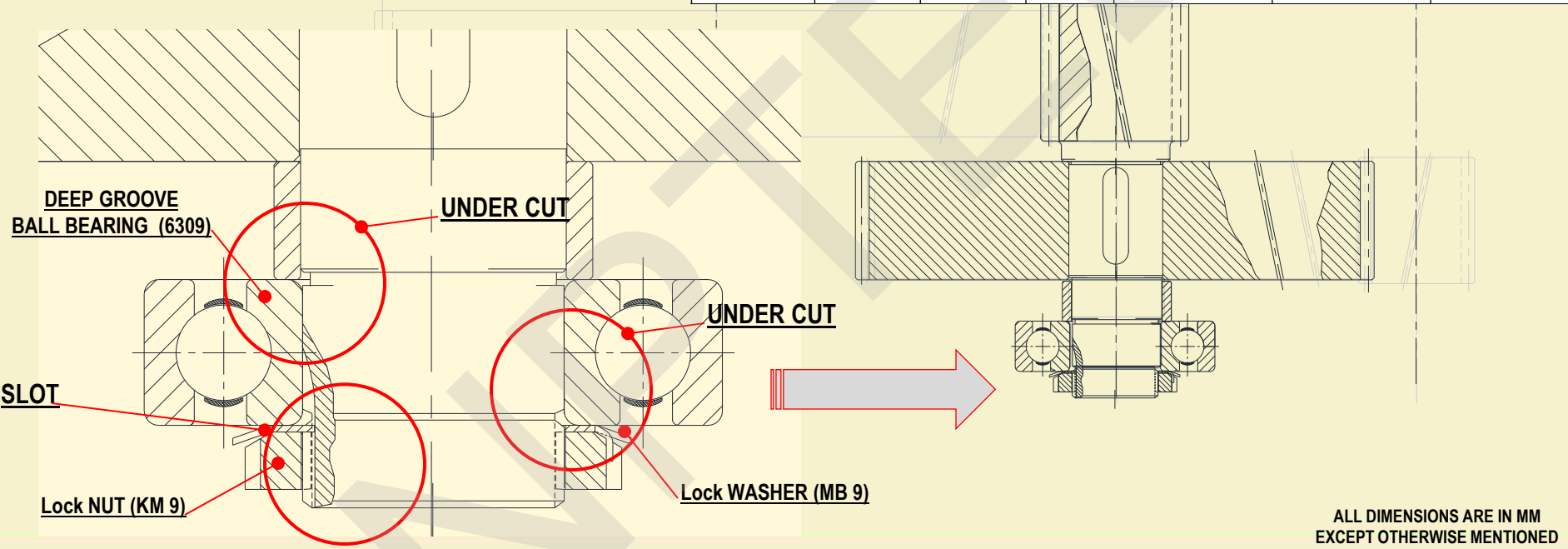


Bearing No.	Inner Dia. (d)	Outer Dia. (D)	Width (B)	Corner Radius (r) Approx.	Basic Load Capacity	
	mm	mm	mm	mm	Dynamic (C)	Static (C _o)
	mm	mm	mm	mm	Newton	Newton
6309	45	100	25	2.5	40130	29200

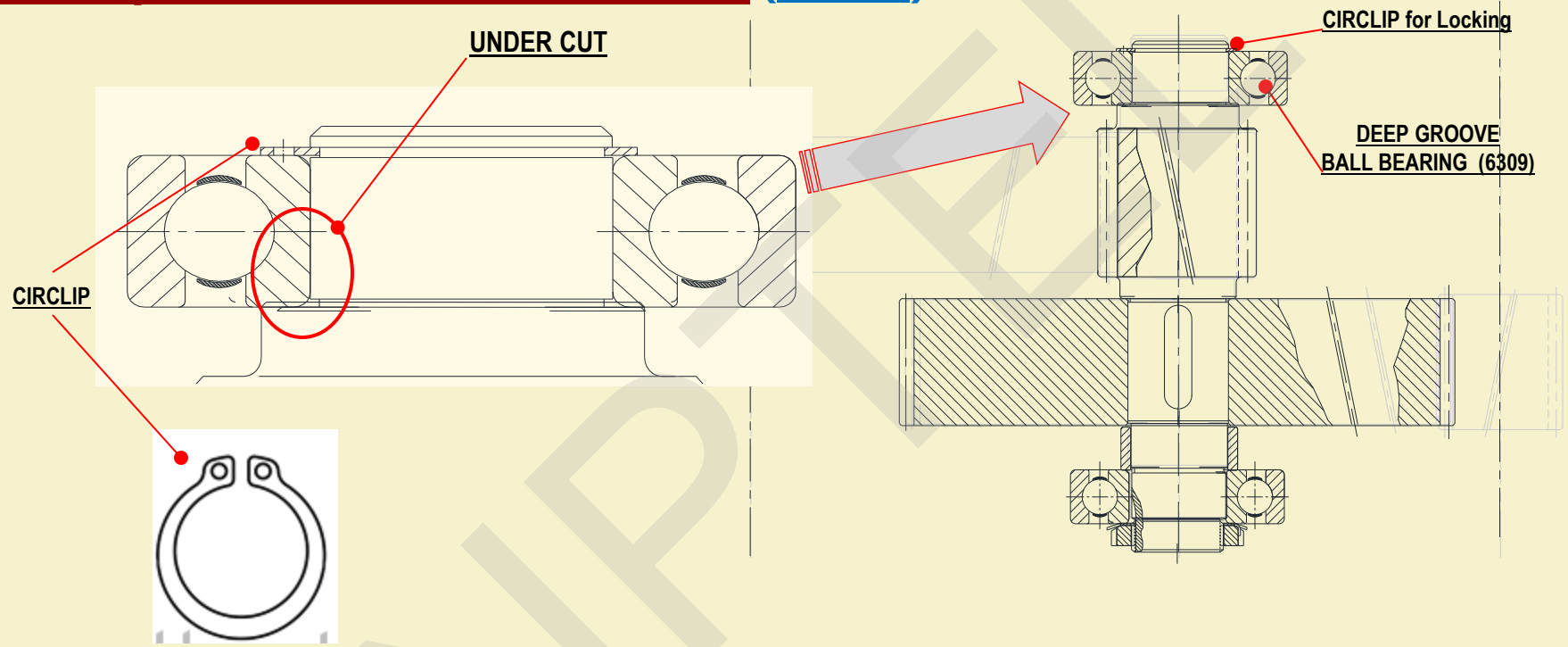
ALL DIMENSIONS ARE IN MM
EXCEPT OTHERWISE MENTIONED

Development of Intermediate Shaft (Contd...) : Details for Gear End Bearing Locking with Shaft :

Bearing No.	Inner Dia. (d)	Outer Dia. (D)	Width (B)	Corner Radius (r) Approx.	Basic Load Capacity	
					Dynamic (C)	Static (C ₀)
	mm	mm	mm	mm	Newton	Newton
6309	45	100	25	2.5	40130	29200



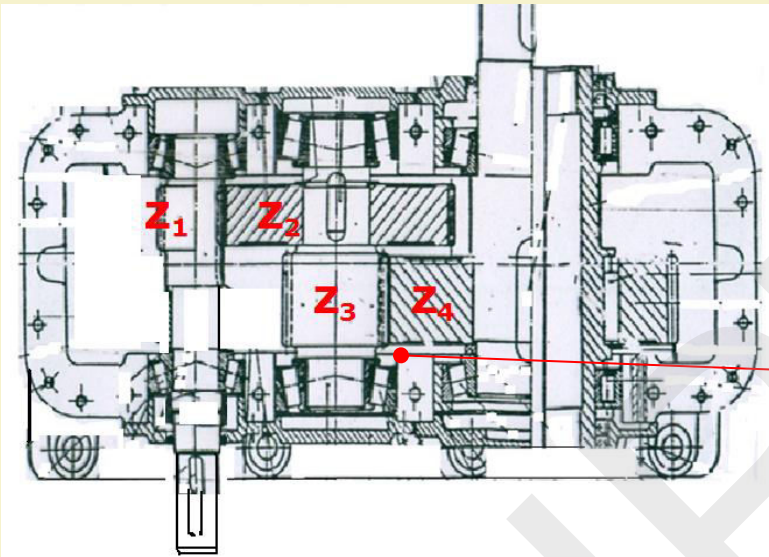
Development of Intermediate Shaft (Contd...) : Details of Pinion End Bearing Locking with Shaft :



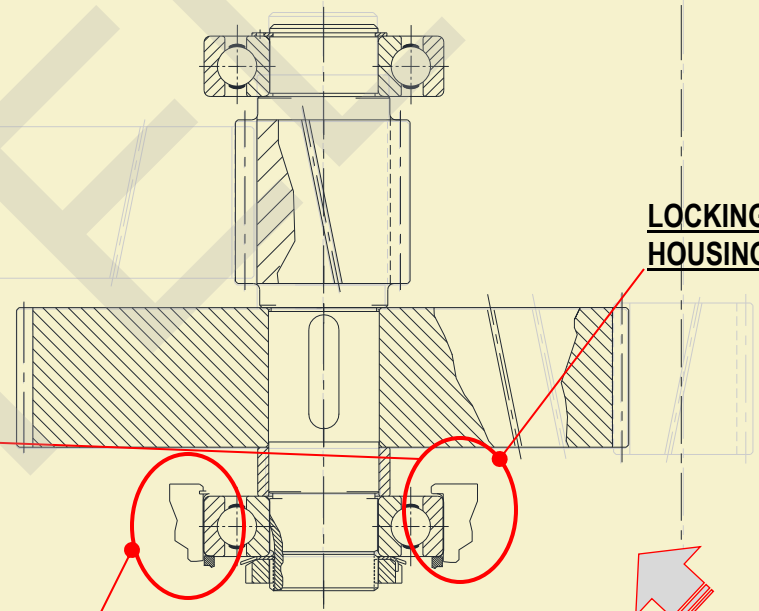
ALL DIMENSIONS ARE IN MM
EXCEPT OTHERWISE MENTIONED

Development of Intermediate Shaft (Contd...) :

Details of Locking with Housing :



Assembled plan view of a 2-stage gear box.



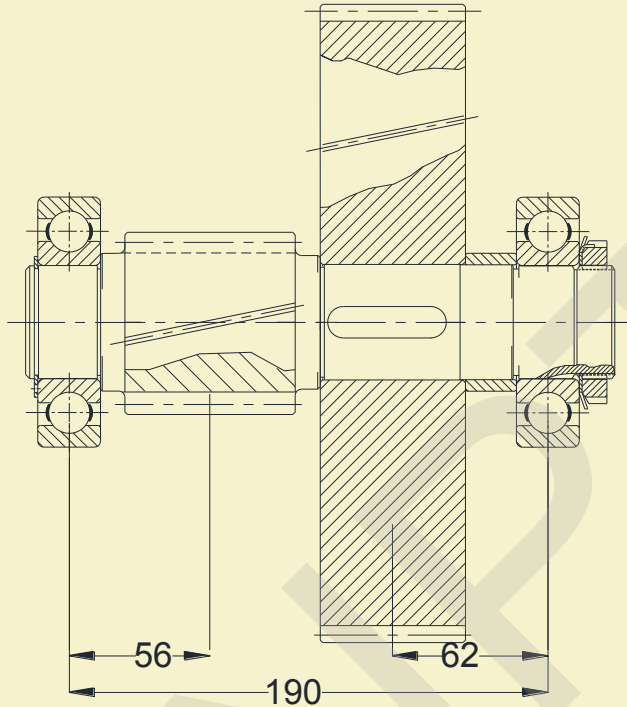
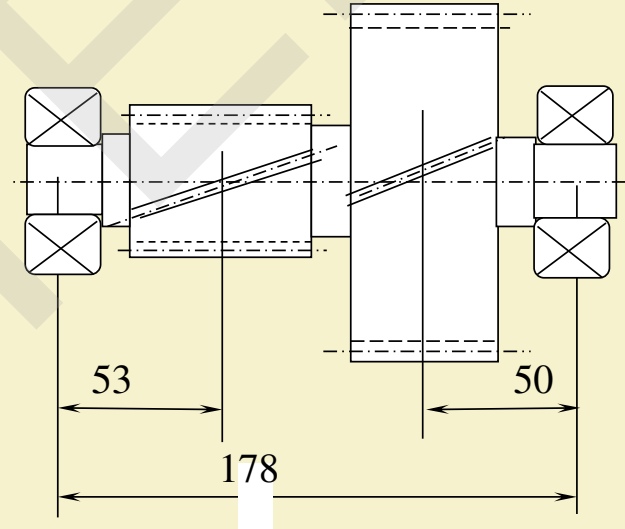
LOCKING WITH HOUSING (CIRCLIP)

LOCKING WITH HOUSING

SUB ASSEMBLY OF INTERMEDIATE SHAFT

ALL DIMENSIONS ARE IN MM EXCEPT OTHERWISE MENTIONED

Development of Intermediate Shaft (Contd...) : Details of Sub assembly of Intermediate Shaft :



**SUB ASSEMBLY OF
INTERMEDIATE SHAFT**

ALL DIMENSIONS ARE IN MM
EXCEPT OTHERWISE MENTIONED

Thank You



IIT KHARAGPUR



NPTEL ONLINE
CERTIFICATION COURSES

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